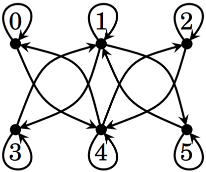
Relations

March 24, 2015

11.0 Q 4:Here is a diagram for a relation R on a set A. Write sets A and R.



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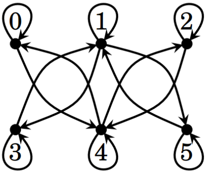

From the diagram above we can conclude R=
$\{(0,0),(0,4),(4,0),(1,1),(1,3),(3,1),(2,2),(2,4),(1,5),(5,1),(3,3),(4,2),(4,4),(5,5)\}$

A=$\{0,1,2,3,4,5\}$

11.0 Q 8 Let A=$\{1,2,3,4,5,6\}$. Observe that $\phi \subset$ A X A so R= ϕ is a relation on A. Draw a diagram for this relation.

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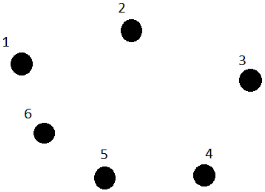

11.1 Q 2:Consider The relation $R=\{(a,b),(a,c),(c,c),(b,b),(c,b)\}$ on the set $A=\{a,b,c\}$. Is R reflexive? Symmetric? Transitive? If the property does not hold , say why?

11.1 Q 2:Consider The relation R={(a,b),(a,c),(c,c),(b,b),(c,b)} on the set A={a,b,c}. Is R reflexive? Symmetric? Transitive? If the property does not hold , say why?
The above relation does not contain (a,a), so it is not reflexive over A.
Also it contains (a,c) but not (c,a), hence not symmetric.
Transitivity however exist here as for (a,b) and (b,b) we have (a,b) in set R; for (a,c) and (c,b), we have (a,b) in R. Also for (a,c) and (c,c) we have (a,c) in R.

11.1 Q 8: . Define a relation on $\mathbb{Z}$ as xRy if $\|x - y\| < 1$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why. What familiar relation is this?
For the absolute value of a difference to be less than 1 , the value of x and y has to be same. So R is reflexive, symmetric and transitive.
Another familiar relation is x=y

11.1 Q 10: Suppose A $\neq \phi$;. Since $\phi \subset$ A X A, the set R $= \phi$ is a relation on A. Is R reflexive?Symmetric? Transitive? If a property does not hold, say why.
Solution: The relation is valid.
Let A= $\{$a,b,c$\}$;
since (a,a),(b,b),(c,c) $\notin \phi$ So the relation is not reflexive.
There is no relation between a and b and b and a (hence symmetric). Also there is no relation between a and b , b and c and thus a and c,thus transitive.

11.2 Q 4: Let A $=\{$a,b, c,d, e$\}$. Suppose R is an equivalence relation on A. Suppose also that aRd and bRc, eRa and cRe. How many equivalence classes does R have?

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Since R is equivalence in A so it has to be reflexive and thus must contain (a,a), (b,b), (c,c), (d,d), (e,e) Also (a,d), (b,c) ,(e,a), (c,e) is in the set R.
Let us consider the sets containing b , (b,c) is a relation. Since cRe so bRe is also an element in the relation (By transitivity) Similarly eRa implies bRa ; aRd implies bRd. Also (b,b) is an element . So we can say b is related to every element of A. Similarly we can say that every element will be related to every element of A. Thus the equivalence class will contain $\{$a,b,c,d,e$\}$.
Alternately, we can answer the question differently as follows. aRd implies that a and d belong to the same class. Similarly, bRc and cRe imply that c and e belong to the same class. Since there is an arc (e, a) from $\{c, e\}$ to $\{a, d\}$, therefore $\{a, b, c, d, e\}$ is one equivalence class.

11.2 Q 6:There are five different equivalence relations on the set A $=\{$a,b, c$\}$. Describe them all. Diagrams will suffice.

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1. $\{$(a,a), (b,b), (c,c)$\}$
2. $\{$(a,a),(a,b)(b,a),(b,b),(c,c)$\}$
3. $\{$(a,a),(a,c),(c,a),(b,b),(c,c)$\}$
4. $\{$(a,a),(b,c)(c,b),(b,b),(c,c)$\}$
5. $\{$(a,a),(a,b),(b,a),(b,b),(c,c),(b,c),(c,b),(a,c),(c,a)$\}$

11.2 Q 12: Prove or disprove: If R and S are two equivalence relations on a set A, then $R \cup S$ is also an equivalence relation on A.

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Let R= $\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$
S=$\{(1,1,),(2,2),(3,3),(2,3),(3,2)\}$
$R \cup S$= $\{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$.Here (1,2) and (2,3) $\in R \cup S$ but here (1,3) $\notin R \cup S$ thus the transitive law is violated. From the above, we see that $R \cup S$ is not a equivalence relation.

11.3 Q 2:List all the partitions of the set A $=\{$a,b, c$\}$. Compare your answer to the answer to Exercise 6 of Section 11.2.

11.3 Q 2:List all the partitions of the set A $=\{a,b, c\}$. Compare your answer to the answer to Exercise 6 of Section 11.2.

The possible partitions are :

1. $\{a\}$, $\{b\}$, $\{c\}$
2. $\{a,b\}$, $\{c\}$
3. $\{b,c\}$, $\{a\}$
4. $\{a,c\}$, $\{b\}$
5. $\{a,b,c\}$