Proofs using contrapositive and contradiction methods)

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Use the method of contrapositive proof to prove the following :

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Proof

- Equivalent contrapositive statement: **if a divides b, a divides bc**.
- Suppose a divides b.
- So a divides any multiple of b
- Thus a divides bc, c an integer.
- The contrapositive statement is true.
- So **if** a **doesn't divide** bc **is true , then** a **doesn't divide** b is also true.

Q 12: Suppose a $\in \mathbb{Z}$, **If a^2 is not divisible by 4 , then a is odd**.

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Proof

- Equivalent contrapositive statement: **if a is even, then a^2 is divisible by 4.**
- Suppose a is even.
- So $a = 2k$, k is an integer.
- Thus $a^2 = 4k^2$
- So a^2 is divisible by 4.
- The contrapositive statement is true.
- So If a^2 is not divisible by 4 is true , then a is odd is also true.

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Q 24: If $a = b(\text{mod n})$ and $c = d(\text{mod n})$, then $ac = bd(\text{mod n})$.
Proof : **Using Direct proof**
Let $a = nr + b$ and $c = ns + d$, r, s are integers.
So $ac = (nr + b)(ns + d)$.
i.e $ac = (rsn + dr + bs)n + bd$.
Thus $ac = bd(\text{mod n})$.
This completes the proof .

Q 28: If n $\in \mathbb{Z}$ then 4 doesn't divide $(n^2 - 3)$
Proof: bf Using contrapositive proof.
Contrapositive statement: **if 4 divides $(\mathbf{n^2 - 3})$, then n is not an integer.**
$(n^2 - 3) = (n + \sqrt{3})(n - \sqrt{3})$.
In order for 4 to divide $(n^2 - 3)$, n cannot not be an integer. One of the factors must be even.
Thus n $\notin \mathbb{Z}$.
This completes the proof.

Prove by the method of Contradiction .

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Q 4: Prove that $\sqrt[3]{2}$ is irrational.

Proof:

Suppose $\sqrt[3]{2}$ is a rational number such that $\sqrt[3]{2}$ =a/b where a and b are integers having no common factor.($\neg$($\sqrt[3]{2}$ is irrational))

$\Rightarrow 2 = a^3/b^3$

$\Rightarrow 2b^3 = a^3$

$\Rightarrow$ Thus a^3 is even

$\Rightarrow$ thus a is even.

Let $a = 2k$, k is an integer.

So $2b^3 = 8k^3$

$\Rightarrow b^3 = 4k^3$

So b is also even.

But a and b had no common factors. Thus we arrive at a contradiction.

So $\sqrt[3]{2}$ is irrational.

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Proof: **Proof by contradictiction**
Suppose $a^2 + b^2 = c^2$ and a,b are odd.
Let a=2m+1 and b=2n+1 since they are both odd, m and n are integers.

$$a^2 + b^2 = (2m + 1)(2n + 1)$$

i.e. $a^2 + b^2 = 4(m^2 + n^2 + n + m) + 2$.
i.e. $a^2 + b^2 = 4X + 2$, X is an integer.
Now $c^2 - 2 = 4X$
Therefore, c is even
Let $c = 2r$, r is an integer.
So $4r^2 = 2(2X + 1)$
$2r^2 = (2X + 1)$ which is a contradiction (the lhs is even, and the rhs is odd)
Thus if $a^2 + b^2 = c^2$, then a or b is even.

Q 14: If A and B are two sets , then $A \cap (B - A) = \phi$.

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Proof by contradiction

Suppose $A \cap (B - A) \neq \phi$.

Therefore, there exist an $x \in A \cap (B - A)$.

$\Rightarrow x \in A$ and $x \in (B - A)$

$\Rightarrow$ x $\in A$ and x $\in B$ and x $\notin A$

Now we arrive at a contradiction as x cannot both belong and not belong to A

So A $\cap (B - A) = \phi$