

Homework 1
MACM 101
March 20, 2015
Date due: March 27, 2015.

Use Induction principle to prove the following problems. The homework is due at the beginning of the class.

1. Prove the validity of the following Rule of Inference for all integers $n \geq 1$:

$$\begin{array}{ccc} p_1 & \rightarrow & p_2 \\ p_2 & \rightarrow & p_3 \\ & \dots & \dots \\ p_n & \rightarrow & p_{n+1} \\ \hline & \neg p_{n+1} & \\ \hline & \neg p_1 & \end{array}$$

2. Suppose n straight infinite lines lie on a plane in such a way that no two of the lines are parallel, and no three of the lines intersect at a single point. Show that this arrangement divides the plane into $\frac{n^2+n}{2} + 1$ regions.
3. Prove that $3^0 + 3^1 + 3^2 + \dots + 3^n = \frac{3^{n+1}-1}{2}$.
4. Every integer $n \geq 14$ is expressible in the form $5a + 7b + 9c$ where a, b, c are nonnegative integers.
5. Define the following sequence of numbers: $a_1 = 2$ and for $n \geq 2$, $a_n = 5a_{n-1}$. Find a formula for a_n and then prove that its validity.