Set Theory

Tutorial 1

January 17, 2015

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So in set builder form , we can write it as :$\{3x/4 : x \in Z\}$

Section 1.1 (C)

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Hence the cardinality is 4

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Let A= $\{x \in R : x^2 = x\}$ =$\{0,1\}$
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So AXB =$\{(0,1),(1,1)\}$

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So our set should be $\{(w,x,y,z)\text{: } w \in A, x \in A, y \in A, z \in A\}$

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$\{(0,0,0,0),(0,0,0,1),(0,0,1,0),(0,0,1,1),(0,1,0,0),(0,1,0,1),(0,1,1,0),$
$(0,1,1,1),(1,0,0,0),(1,0,0,1),(1,0,1,0),(1,0,1,1),(1,1,0,0),(1,1,0,1),(1,0,1,1)$
$,(1,1,1,1)\}$ is the required set.

section 1.3

Find the subsets of
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Q 14. R^2 is the subset of R^3

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False $R^2 = \{(x, y) : x \in R, y \in R\}$,

$R^3 = \{(x, y, z) : x \in R, y \in R, z \in R\}$

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Q 16. { (x,y) :$x^2 - x = 0\} \subset \{(x, y) : x - 1 = 0\}$

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False $R^2 = \{(x,y) : x \in R, y \in R\}$,
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Q 16. $\{ (x,y) : x^2 - x = 0\} \subset \{(x,y) : x - 1 = 0\}$
As the value of x in the first set is 0 and 1 whereas in the second set it is just 1, so the above statement is **False**

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Q 6: P({1,2})X P({3})

Solution

P({1,2})={ϕ,{1},{2},{1,2}}

P({3})={ϕ,{3}}

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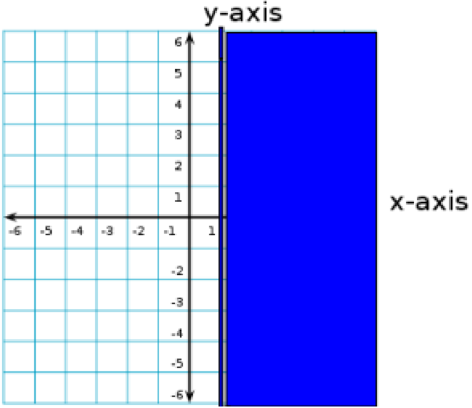
Q 6: P({1,2})X P({3})

Solution

P({1,2})={ϕ,{1},{2},{1,2}}

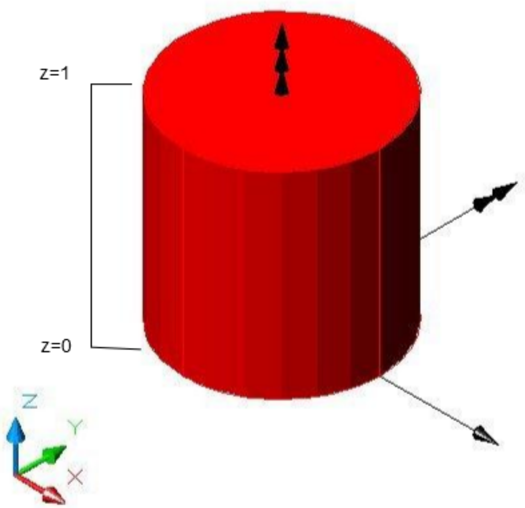
P({3})={ϕ,{3}}

So the required solution is {(ϕ,ϕ),(ϕ,{3}), ({1},{3}), ({1},ϕ), ({2},{3}), ({2},ϕ), ({1,2},ϕ), ({1,2},{3})}

1. **Section 1.1 Q 48.**
   Sketch the graph $\{(x, y) : x, y \in R, y > 1\}$
   Solution
   As y can take any value and x can take only values greater than 1. So the graph can be as follows
   

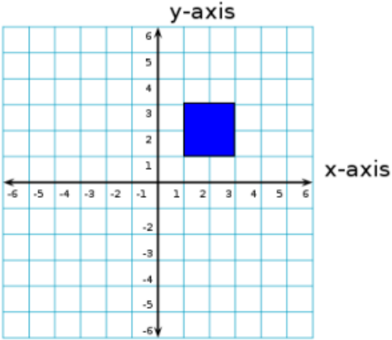
2. **Section 1.2 Q 20**
   Sketch the graph $\{(x, y) \in R^2 : x^2 + y^2 \leq 1\} X [0, 1]$
   Solution

Here the first set represents a circle with radii less than or equal to 1. When we find its cross product with [0,1] , it represents the set $\{(x, y, z) : (x, y) \in R^2 : x^2 + y^2 \leq 1, z \in [0, 1]\}$. So we can say that it gives rise to a solid cylinder with length 1. Thus the resulting graph is

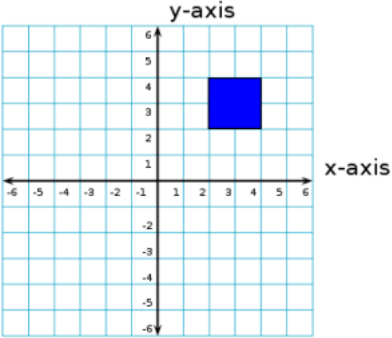


3. **Section 1.5 (5)**
Sketch the sets X=[1,3] X [1,3] and Y=[2,4]X[2,4] on the plane R^2 . On separate drawings shade X $\cup Y, X \cap Y$, X-Y, Y-X
Solution:Since [1,3] contains all points from 1 to 3 so

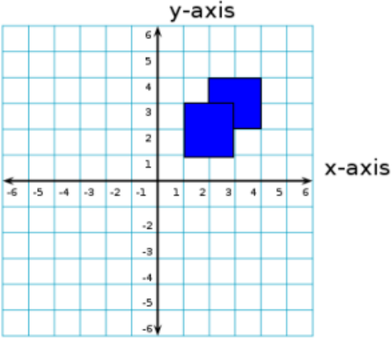
[1,3]X[1,3] means (x,y) such that its x coordinates can take all real values from 1 to 3 and so can y. So the resultant graph is a square with vertices (1,1),(1,3),(3,1),(3,3).



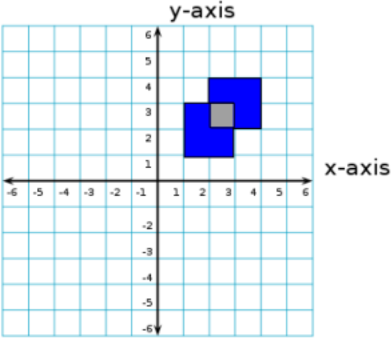
Similarly [2,4]X[2,4] will have the graph as follows:



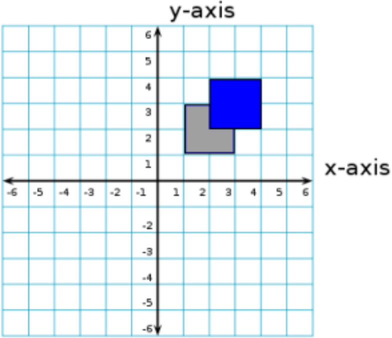
So $X \cup Y$ is denoted by the blue colored portion:



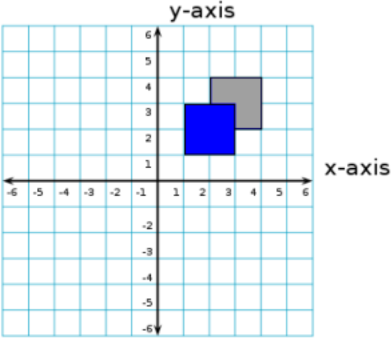
$X \cap Y$ is denoted by the grey portion



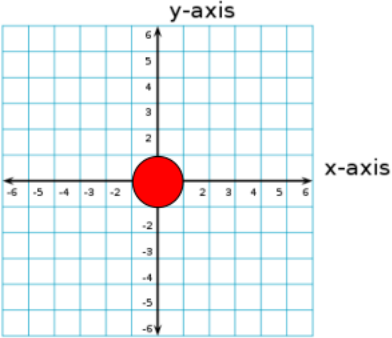
X-Y is denoted by the grey portion.



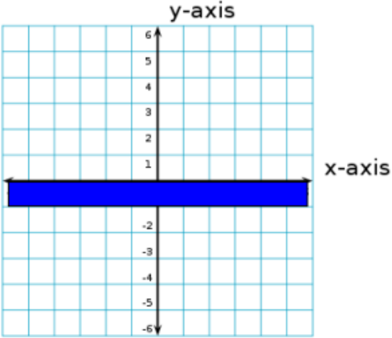
Y-X is denoted by the grey portion



4. **Section 1.5 Q 8**
Sketch the sets X=$\{(x, y) : x, y \in R, X^2 + y^2 \leq 1\}$ and Y=$\{(x, y) : x, y \in R, 0 \leq y \leq -1\}$ on the plane R^2 . On separate drawings shade X $\cup Y, X \cap Y$, X-Y, Y-X
Solution: The graph of X will represent a circle with radius equal to 1



The graph of Y is as follows



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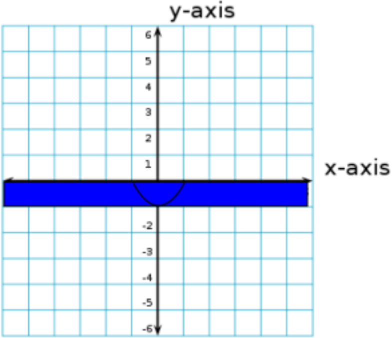

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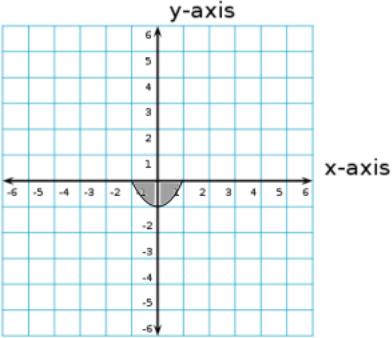

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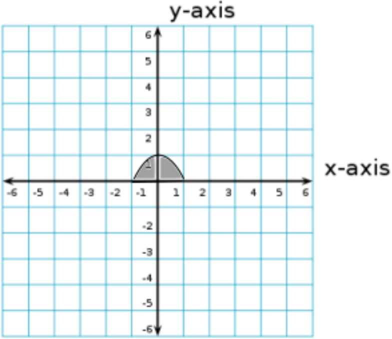

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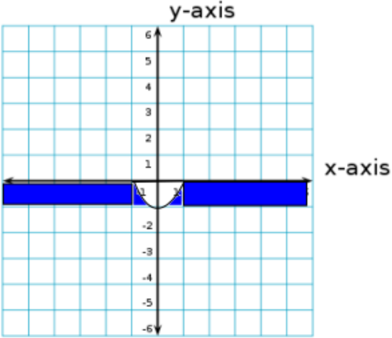
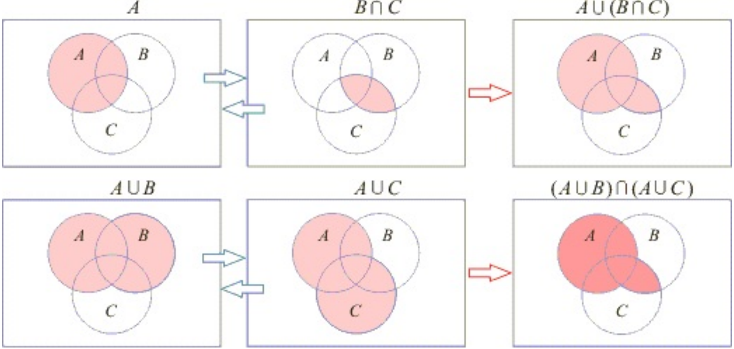


5. **Section 1.7 Q 5.**
   Draw Venn diagrams for $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$. Based on your drawings, do you think $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.?
   Solution



Thus from above we see that the distributive law is true .

Confusions on the following questions during the tutorial are cleared in the text:
Section 1.3 [Q14]R^2 is the subset ofR^3 This is false, the reason is what we discussed in the tutorial
Section 1.8 [Q13] The answer is correct. The union part is true but the intersection part is false. For the intersection part, make a counter example. For example J={1,2}, I{1,2,3}, A1={1,2}, A2={2,3}, A3={1,3} The the intersection of all the J sets(that is

the intersection of A1 and A2) would be $\{2\}$ and the intersection of all the I sets(that is the intersection of A1, A2 and A3) would be empty set. So false.
For the quiz coverage, the professor has already announced in the lecture.