Here are some challenging questions on **relations**. You should solve these as a preparation for the quiz.

1. Determine which of the following relations are reflexive, symmetric, transitive. antisymmetric and transitive. Determine whether the relation is an equivalence relation or a partial order.

   (a) Let A represent the students in MACM 101 class, and let $R = \{(x,y) | x, y \in A \wedge \mid \text{weight}(x) - \text{weight}(y) \mid \leq 10 \; lbs\}$.

   (b) Let R be a relation on $\mathbb{Z} \times \mathbb{Z}$ where $((a,b),(c,d)) \in R$ if and only if $a = c$ and $b \leq d$. Note that $((10,3),(10,10)) \in R$, but $((10,10),(10,3)) \notin R$.

2. Show that the relation R on $\{1,2,3\} \times \{1,2,3\}$ where $((a,b),(c,d)) \in R$ if and only if $(a < c) \vee (a = c \wedge b \leq d)$ is true, is a partial order.
   (Such an order is called the *lexicographic order*.)

3. Let $\Sigma = \{a,b\}$. Then Σ^4 is the set of all strings over Σ of length 4. Define a relation R on Σ^4 where $s,t \in \Sigma^4$ if and only if s and t have the same first two characters. Thus *(abaa,abba)*$\in R$, but *(aabb,bbaa)*$\notin R$. Show that R is an equivalence relation. What are the equivalence classes induced by R?

4. Let $A = \{2,4\}$ and $B = \{6,8,10\}$ and define the binary relations R and S from A to B as follows:

$$(x,y) \in A \times B, (x,y) \in R \Leftrightarrow x | y.$$
$$(x,y) \in A \times B, x \; S \; y \Leftrightarrow y - 4 = x.$$

   List the elements of $A \times B, R, S, R \cup S$, and $R \cap S$.

5. Let R be a binary relation on a set A and suppose that R is symmetric and transitive. Prove the following: If for every $x \in A$, there is a $y \in A$ such that xRy then R is reflexive and hence an equivalence relation on A.