Solutions to Quiz 1
MACM 101

Total Marks: 50
Date: Week 3

1. (5 points) Write the following set by listing their elements between braces.

   (a) $\{x \in \mathbb{Z} : -3 < x \leq 2\}$
   {-2,-1,0,1,2}

   (b) $\{x \in \mathbb{R} : sinx = 0\}$
   $\{\ldots, -2\pi, \pi, 0, \pi, 2\pi, \ldots\}$

   (c) $\{x \in \mathbb{R} : sin\pi x = 0\}$
   $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

   (d) $\{x \in \mathbb{R} : x^2 = 7\}$
   $\{-\sqrt{7}, \sqrt{7}\}$

   (e) $\{x \in \mathbb{Z} : \mid 2x \mid < 5\}$
   $\{-2, -1, 0, 1, 2\}$

   (f) $\{x \in \mathbb{Z} : -2 < x \leq 7\}$
   $\{-1, 0, 1, 2, 3, 4, 5, 6, 7\}$

   (g) $\{x \in \mathbb{Z} : -5 < x \leq 2\}$
   $\{-4, -3, -2, -1, 0, 1, 2\}$

2. (5 points) Write the following set in set-builder notation.

   (a) $\{0, 1, 4, 9, \ldots\}$
   $\{x^2 : x \in \mathbb{Z}\}$

   (b) $\{2, 3, 5, 7, 11, \ldots\}$
   $\{x :$ x is a prime number$\}$

   (c) $\{3, 4, 5, 6, 7, 8\}$
   $\{x : -3 \leq x \leq 8\}$

   (d) $\{-3, -2, -1, 0, 2, 3\}$
   $\{x : -3 \leq x \leq 3 \text{ and } x \neq 1\}$

(e) $\{0, 1, 8, 27, 64, 125, \ldots\}$
$\{x^3 : x \in \mathbb{Z}^+\}$

(f) $\{0, -1, -4, -9, \ldots\}$
$\{-x^2 : x \in \mathbb{Z}\}$

3. (10 points) Suppose $A = \{0, 1\}$ and $B = \{1, 2\}$. Find
$\mathscr{P}(A) = \{\phi, \{0\}, \{1\}, \{0, 1\}\}$
$\mathscr{P}(B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

   - $\mathscr{P}(A) \cap \mathscr{P}(B)$.
     $\{\phi, \{1\}\}$
   - $\mathscr{P}(A \cap B)$.
     $\{\phi, \{1\}\}$
   - $\mathscr{P}(A) - \mathscr{P}(B)$.
     $\{\{0\}, \{1, 0\}\}$

4. (10 points) Let $\mathbb{R}$ be the universal set. Let $A = \{1\}$, $B = (0, 1) = \{x : 0 < x < 1\}$
and $C = [0, 1] = \{x : 0 \leq x \leq 1]$. Write down the following sets.

   - $A \cup B$; $A \cap B$; $B \cap C$; $A \cup C$; $A \cap C$
     (0,1] ϕ (0,1) [0,1] {1}

   Are any of the pairs of sets A, B and C disjoint? A and B are disjoint.

5. (10 points) Let A, B and C be three arbitrary subsets of the universal set U. Use an element containment proof (i.e. prove that the left side is a subset of the right side and the right side is a subset of the left side) to prove the following:

   - $\overline{A \cup B} = \overline{A} \cap \overline{B}$.
     Let $x \in \overline{A \cup B}$
     $\Rightarrow x \notin A \cup B$
     $\Rightarrow x \notin A$ and $x \notin B$
     $\Rightarrow x \in \overline{A}$ and $x \in \overline{B}$
     $\Rightarrow x \in \overline{A} \cap \overline{B}$
     So $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$.

     Let $x \in \overline{A} \cap \overline{B}$

$\Rightarrow x \in \overline{A}$ and $x \in \overline{B}$
$\Rightarrow x \notin A$ or $x \notin B$
$\Rightarrow x \notin A \cup B$
$\Rightarrow x \in \overline{A \cup B}$
So $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$
Since $\overline{A} \cap \overline{B}$ and $\overline{A \cup B}$ are both subsets of one another , so $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

- $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$.
  Proved in the same way as above.

6. (10 points) Use the membership table method to determine which membership $\subseteq, =, \supseteq$ is true for the following pair of sets.

- $(A - B) \cup (A - C), \quad A - (B \cap C)$.

A	B	C	(A-B)	(B-C)	(A-C)	A- (B ∩ C)	(A- B) ∪ (A-C)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	0	1	0	0	0
0	1	1	0	0	0	0	0
1	0	0	1	0	1	1	1
1	0	1	1	0	0	1	1
1	1	0	0	1	1	1	1
1	1	1	0	0	0	0	0

From above , we see that A-(B∩C) and $(A - C) \cup (B - C)$ are equal .

- $(A - C) - (B - C), \quad A - B$.

A	B	C	(A-B)	(B-C)	(A-C)	(A-C)-(B-C)
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	1	0	0
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	0	1	1	0	0	0

From above table we see that (A-C)-(B-C) is the subset of (A-B) .

- $(B - C), \quad (B - A) - (C - A)$

7. (Bonus) (10 points) Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
Let $(x, y) \in A \times (B \cap C)$
$\Rightarrow x \in A, y \in B \cap C$

$\Rightarrow x \in A, y \in B$ and $y \in C$
$\Rightarrow x \in A, y \in B$ and $x \in A, y \in C$
$\Rightarrow (x,y) \in A \times B$ and $(x,y) \in A \times C$
$\Rightarrow (x,y) \in (A \times B) \cap (A \times C)$
So $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$.

Similarly let $(x,y) \in (A \times B) \cap (A \times C)$
$\Rightarrow (x,y) \in A \times B$ and $(x,y) \in A \times C$
$\Rightarrow x \in A, y \in B$ and $x \in A, y \in C$
$\Rightarrow x \in A, y \in B$ and $y \in C$
$\Rightarrow x \in A, y \in B \cap C$
$\Rightarrow (x,y) \in A \times (B \cap C)$
So $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$.
Since are both subsets of one another , so $A \times (B \cap C) = (A \times B) \cap (A \times C)$.