Logic

Tutorial 2

February 1, 2015

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P: There is a quiz scheduled for Wednesday.

Q: There is a quiz scheduled for Friday.

Hence, the answer is $P \vee Q$

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Solution:
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Q: $5a \in \mathbb{Q}$
Tips: P and Q are interchangeable in “if and only if” case.

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P	Q	$P \vee Q$	$\sim (P \vee Q)$	$\sim P$	$\sim (P \vee Q) \vee \sim P$
T	T	T	F	F	F
T	F	T	F	F	F
F	T	T	F	T	T
F	F	F	T	T	T

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$(R \vee S)$ is false if and only if R is false and S is false.

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$(R \vee S)$ is false if and only if R is false and S is false.

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Hence, P is true and Q is true.

In summary, the answer is that P is true, Q is true, R is false and S is false.

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Use truth tables to show that the following statements are logically equivalent

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Solution

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
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Solution

If X belongs to the power set of natural number, then X is a real number. This can also be written a $\forall X \in \mathbb{P}(\mathbb{N}) \Rightarrow X \in \mathbb{R}$.

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Q4: $\forall X \in \mathbb{P}(\mathbb{N}), X \in \mathbb{R}$.
Solution
If X belongs to the power set of natural number, then X is a real number. This can also be written a $\forall X \in \mathbb{P}(\mathbb{N}) \Rightarrow X \in \mathbb{R}$.
False, because X is a set, not a real number.

Translate each of the following sentences into symbolic logic.
Q8: I don't eat anything that has a face.

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Solution
Suppose F is a universal set .
$f(x)$: $x \in F$, x has a face.
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Solution
Suppose F is a universal set .
$f(x)$: $x \in F$, x has a face.
$p(x)$: I eat x.
$\forall x \in F, f(x) \to \neg p(x)$
$\Leftrightarrow \forall x \in F, (\neg f(x) \vee \neg p(x))$

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Solution

$p(x) : x \in R$.
$q(a, x) : a + x = x, a, x \in R$

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Q6: There exists a real number a for which $a + x = x$ for every real number x.

Solution

$p(x) : x \in R.$

$q(a, x) : a + x = x, a, x \in R$

$\exists a \in R, \forall x \in R, q(a, x)$