Solutions to the assign even-numbered questions of Chapter 3

3.1(2) We will assume that the letters of a 3-letter code need not be distinct. In this case, the answer is 26^3 since the order of letters in a code is important.

3.1(4) Here order is important. Repetition is not allowed. Five cards lined up in a row is a length-5 list. Thus the answer is $C(4,1) \times (13.12.11.10.9) = C(4,1) \times \frac{13!}{8!}$. $C(4,1)$ choices are there for the suit.

3.1(8)(a) The answer is the number of length-5 lists with repetitions minus the number of length-5 lists with no repetitions. Therefore, $5^5 - 5!$ is the answer.

3.1(8)(b) Since for a length-6 list, there are 5 objects to choose from, there will always be a letter which is repeated. Hence the answer is 5^6.

3.1(12) Let A_S be the set of lists that end in an S, repetition allowed. Let A_S^0 be the set of lists that have no O and end in an S. Let A_S^1 be the set of lists that have one O and end in an S. Now $|A_S| = 5^5$, $|A_S^0| = 4^5$, and $|A_S^1| = C(5,1) \times 4^4$. $C(5,1)$ is the number of choices for one O to be placed. Thus the answer is $|A_S| - |A_S^0| - A_S^1|$. Note that A_S^0 and A_S^1 are disjoint.

3.3(10) The numbers of ways to select 3 men and 2 women are $C(5,3)$ and $C(7,2)$ respectively. Therefore the number ways to select a committee is $C(5,3) \times C(7,2)$.

3.3(12) There are $C(21,10)$ ways to select the red team. Once the red team is selected, there is only one way to select the blue team. Thus the answer is $C(21,10)$.

3.4(6) Fact 1.3 says that a set of size n realizes 2^n subsets. Definition 3.2 says that $\binom{n}{k}$ is the number of ways to chose a size-k subset. Thus
$2^n = \sum_{i=0}^{i=n}$ **number of subsets of size** i. Therefore, $2^n = \sum_{i=0}^{i=n} \binom{n}{i}$.

3.4(8) We need to show that $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n-1}{k-1}$. Use the definition of factorials to show this identity.

3.5(4)(a) Let set A_T contains lists that begin with T. Let set A_Y contains lists that end with Y. Let A_{TY} contains lists that begin with T and end with Y. There are 6^4 possible ways to select length-4 lists. Therefore, the answer is $6^4 - |A_S| - |A_Y| + |A_{TY}|$. We applied the principle inclusion and exclusion.

3.5(4)(b) In this case T of THE can occupy positions one or two only. The remaining position can be filled in 5 ways. Thus the answer is $2 * 5$.

3.5(4)(c) In this case the answer is $3 * 5^2$

3.5(6) This is false. $A_1 = \{a, b\}, A_2 = \{b, c\}, A_3 = \{a, c\}$.

3.5(8) Number of ways to deal 4 cards where all 4 cards are of different suites = $C(13, 1)^4$. Number of ways to select 4 red cards = $C(26, 4)$. The answer is therefore $C(13, 1)^4 + C(26, 4)$.