Test bank questions on set theory

1. Prove or disprove: if A, B and C are sets, then $A - (B \cap C) = (A - B) \cap C$.

2. Prove or disprove $A \triangle (B \triangle C) = (A \triangle B) \triangle C$.

3. Determine which relationship $\subseteq, =, \supseteq$, is true for the pair of sets.

   (a) $A \cup (B \cap C), (A \cup B) \cap C$

   (b) $(A - B) \cup (A - C), A - (B \cap C)$.

   (c) $(A - C) - (B - C), A - B$.

4. Determine whether the given set is the power set of some set. If the set is a power set, give the set of which it is a power set.

   (a) $\{\Phi, \{\Phi\}, \{a\}, \{\{a\}\}, \{\{\{a\}\}\}, \{\Phi, a\}, \{\Phi, \{\{a\}\}\},$
   $\{a, \{a\}\}, \{a, \{\{a\}\}\}, \{\{a\}, \{\{a\}\}\}, \{\Phi, a, \{a\}\},$
   $\{\Phi, a, \{\{a\}\}\}, \{\Phi, \{a\}, \{\{a\}\}\}, \{a, \{a\}, \{\{a\}\}\},$
   $\{\Phi, a, \{a\}, \{\{a\}\}\}\}$.

5. Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$ for all sets A and B.

6. True or False? Assume that the statement applies to all sets.

   (a) There is a set A such that $|\mathscr{P}(A)| = 12$.

   (b) $A \triangle A = A$.

   (c) $\overline{(A \cup \overline{B})} \cup \overline{A} = \overline{A}$.

7. Find three subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ such that the intersection of any two has size 2 and the intersection of all three has size 1.

8. Fill the blanks

   (a) $\bigcup_{i=1}^{+\infty} [-\frac{1}{i}, \frac{1}{i}] = \ldots\ldots\ldots$

   (b) If $U = \{1, 2, ..., 9\}$, A = all multiples of 2, B= all multiples of 3, and $C = \{3, 4, 5, 6, 7\}$, then $C - (B - A) = \ldots\ldots\ldots$

9. Suppose A $\{a, b, c\}$. TRUE or FALSE?

   (a) $\{a, c\} \subset A$.
   (b) $\{a, c\} \in A \times A$.
   (c) $\Phi \subseteq A \times A$.

10. Determine if the set is finite or infinite. If finite, find its size.

   (a) $A \times A$ where $A = \{x : x \in \mathbb{Z} \text{ and } x^2 < 10\}$.
   (b) $A \times A$ where $A = \{x : x \in \mathbb{Q} \text{ and } x^2 < 10\}$.
   (c) $\{x : x \in \mathbb{N} \text{ and } 9x^2 - 1 = 0\}$.