

# MACM 101 (Discrete Mathematics I)

Quiz 3: November 2, 2011

Answer all questions.

1. Give a direct proof and an indirect (contraposition) proof of the following:  
For all integers  $k$  and  $l$ , if  $k$  and  $l$  are both even, then  $k + l$  is even.
2. Consider the experiment of throwing two 10-sided dice (indistinguishable) where the faces of a die are labeled from 1 through 10.
  - (a) What is the sample space of the experiment?
  - (b) Consider the event  $E$  where the sum of the faces of the two dice is even. Write down the elements of  $E$ .
  - (c) What is the probability of the event  $E$  occurring?
  - (d) What is the probability of event  $E$  not occurring?
3. (Induction) Prove by induction that  $n * (n - 1) * (n - 2) * ... * 2 * 1 > 2^n$  for all integers  $n > 4$ .
4. There must be something wrong with the following induction proof. What is it?  
*Theorem:* For all positive integers  $n$ ,  $2^{n-1} = 1$ .  
*Proof.* If  $n = 1$ ,  $2^{n-1} = 2^{1-1} = 2^0 = 1$ . Suppose that the theorem is true for all  $n \leq k$ . Now we have

$$2^{(k+1)-1} = 2^k = \frac{2^{k-1} \cdot 2^{k-1}}{2^{k-2}} = \frac{1 \cdot 1}{1} = 1.$$

Therefore, the theorem is true for  $n = k + 1$  as well. Hence the theorem is true for all positive integers (Using the principle of strong mathematical induction) .

5. A market survey of  $n$  people concerning the preference for three brands of detergents, say brand  $A$ , brand  $B$  and brand  $C$ , results in the following statistics. 26 people like brand  $A$ ; 22 people like brand  $B$ ; 34 people like brand  $C$ ; 11 people like brands  $A$  and  $B$ ; 19 people like brands  $A$  and  $C$ ; 14 people like brands  $B$  and  $C$ ; and 9 people like all the brands. What is the value of  $n$ ?

6. Let  $A$  and  $B$  and  $C$  be subsets of the set of all integers, defined by

$$\begin{aligned}A &= \{x \mid 0 < x < 5\} \\B &= \{3, 7, 19, 25\} \\C &= \{1, 3, 5, 7, 11, 13, 17, 19, 23, 29\}\end{aligned}$$

- (a) What is  $A \cup B$ ?
- (b) What is  $B \cap C$ ?
- (c) What is  $A - B$ ?
- (d) What is  $A \cup (B - C)$ ?
- (e) Draw a Venn diagram of the sets  $A, B, C$ , including all elements of these sets and also include in your diagram some elements in  $Z$  which are not in these sets.