Homework 8

MACM 101

April 2, 2014

Date due: April 09, 2014.

PART A: Practice questions. Some of the questions in this part will be covered in the tutorials. You are strongly encouraged to work on these problems. You are not required to hand in the solutions to the problems in this part.

1. Problem from the text:
   (a) Section 7.3: 8, 14, 16
   (b) Section 7.4: 6,7, 16
2. Suppose that $h : X \to Y$ is any one-to-one function, and $g : Y \to Z$ is any onto function. Prove or disprove
   (a) $g \circ h$ must be onto.
   (b) $g \circ h$ must be one-to-one.
   (c) The relation R on Y defined as follows is an equivalence relation: $\forall\, y_1 \in Y\ \forall\, y_2 \in Y\ \ [(y_1, y_2) \in R \Leftrightarrow g(y_1) = g(y_2)]$.
3. Show that there is exactly one smallest (minimum) element of a poset, if such an element exists.
4. Determine all equivalence relation relations on the 3-element set $A = \{a, b, c\}$. For each of these determine the number of ordered pairs in the relation. **Hints:** Equivalence relations partitions A into subsets. There are three cases: (a) Partition A into one part; (b) Partition A into two parts of cardinalities 1 and 2. There are $\binom{3}{1}$ way to select this partition; (c) Partition A into three parts of each of cardinality one. There is just one way to partition this.

Part B: Homework questions.

1. Let R_1 and R_2 be relations on a set A, represented by matrices:

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Showing all your work, determine the matrices representing each of the following relations:

(a) $R_1 \cup R_2$

(b) $R_1 \cap R_2$

(c) $R_1 \circ R_2$

(d) $R_1 \oplus R_2$

2. Give an example of each of the following, if one exists. If none exists, prove the fact.

   (a) a non-empty binary realtion which is both symmetric and antisymmetric.

   (b) a partial ordering R on N such that $R = R^{-1}$.

   (c) an equivalence relation on $\{1, 2, 3, 4, 5, 6\}$ whose equivalence classes are $\{1, 2\}, \{3\}, \{4, 5, 6\}$.

   (d) a partial ordering R on a set S containing elements x and y such that x is minimal and y is minimum, and $x \neq y$.

3. Determine all the equivalence relations R on A where $A = \{1, 2, 3\}$. For each of these, list all ordered pairs in relation R or draw the graph.

4. If the following statement is true, prove it; if it is false, providecounterexamples.

   (a) On the set $\mathbb{R}$, the relation R defined by $(x, y) \in R \Leftrightarrow x \neq y$ is an equivalence relation.

   (b) On the set $\mathbb{Z}$ of all integers, the relation S defined by $(a, b) \in S \Leftrightarrow a|b$ is a partial ordering.

5. Show that there is exactly one greatest (maximum) element of a poset, if such an element exists.