MACM 101 (Homework 4)
February 24, 2014
Date Due: March 3, 2014.

Part A: Practice questions. You are strongly encouraged to work on these problems. You are not required to hand in the solutions to these problems.

1. Problems from the text.
   (a) Section 3.1: $8, 10, 17, 18$
   (b) Section 3.2: $6(bc), 8, 14(c)$
   (c) Section 3.3: $4, 6$
   (d) Section 3.4: $6, 8, 10, 16$
2. Use the method of contrapositive proof to prove the following statements. (Try first the direct proof. You will notice that contrapositive proof is easier.)
   (a) $\forall a, b \in Z$: If $a^2(b^2 - 2b)$ is odd, then a and b are odd.
   (b) $\forall a, b, c \in Z$: If a does not divide bc, then a does not divide b.
3. Use the fact that any odd number is of the form $8k \pm 1$ or $8k \pm 3$ (k is an integer) in order to give a direct proof of the claim that the square of any odd number leaves remainder 1 upon division by 8. Use this to show that 2001 is not the sum of three odd numbers.
4. Let s be a positive integer. Prove that the closed interval $[s, 2s]$ contains a power of 2.
   {Hints: Use proof by cases. Consider two cases: (a) s is a perfect power of 2, and (b) s is not a perfect power of 2, i.e. $2^k < s < 2^{k+1}$ for some integer k.}
5. Suppose A, B and C are subsets of the universe U. Prove that
   (a) if $A \subseteq B$ then $A \cap C \subseteq B \cap C$.
   (b) if $A \subset B$ then $\overline{B} \subseteq \overline{A}$.
   (c) if $A \subset C$ and $B \subseteq C$ then $A \cup B \subseteq C$.
6. Determine if the following statements are true or false:
   (a) A and $B - A$ are disjoint.
   (b) $(A \subseteq \overline{B} \wedge \overline{A} \subseteq \overline{B}) \Rightarrow B = \Phi$.

Part B: Homework questions.

1. Prove, by arguing by contradiction, that $\sqrt{3}$ is an irrational number.

2. Prove, by arguing by contradiction, that there are no integers, a, b, c, d such that

$$x^4 + 2x^2 + 2x + 2 = (x^2 + ax + b)(x^2 + cx + d)$$

3. Use the method of contrapositive proof to prove the following statements. (Try first the direct proof. You will notice that contrapositive proof is easier.)
   (a) $\forall n \in Z$. If n^2 is even, then n is even.
   (b) $\forall x \in R$: If $x^3 - x > 0$ then $x > -1$.
4. Describe each of the following sets in the format $\{x|\text{property of x}\}$.
   (a) $A = \{0, 2, 4, 6, 8, ...\}$
   (b) $B = \{1, 5, 9, 13, ...\}$
   (c) $C = \{1, 1/2, 1/3, 1/4, ...\}$
5. Let U be the universal set and let $A, B, C \subseteq U$. Using the properties of union, intersection and complement and known set laws, simplify the followin:
   (a) $\overline{A \cap U} \cup \overline{A}$
   (b) $(A \cap B) \cap \overline{A}$.