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Midterm 1 (MACM101-D1)
February 3, 2014.
Test duration: 50 minutes

There are eight questions in this test. Answer questions worth 60 points.

1. (10 points) Consider selecting 3 objects from the set $A = \{1, 2, 3, 4, 5, 6\}$.
   (a) How many ordered sequences without repetition can be chosen from A?
   ans= P(6,3)
   (b) How many ordered sequences with repetition can be chosen from A?
   ans= 6^3**; there are 6 choices for each position.**
   (c) How many unordered sequences without repetition can be chosen from A? **ans= C(6,3)**
   (d) How many unordered sequences with repetition can be chosen from A?
   $\binom{6+3-1}{6-1}$**; this r-combinations with repetitions.**
   (e) How many strictly increasing sequences can be chosen from A?
   $\{ < 2, 4, 4 >$ is not a strictly increasing sequence.$\}$
   It is the same as the number of $3-$**combinations without repetitions, since every such** $3-$**element combination, there is only one strictly increasing sequence. Hence the answer is C(6,3).**

2. (10 points) Consider a seven letter word *aeemrry*.
   (a) How many different arrangements of these seven letters are there?
   no constraint: $\frac{7!}{2!2!}$
   (b) How many such arrangements are there that contain *eye*?
   Arrangements with eye: use $\{eye, a, r, m, r\} : \frac{5!}{2!}$
   (c) How many such arrangements are there that contain *eye* and *ram*?
   Arrangements with eye and ram: use $\{ram, eye, r\} : 3!$
   (d) How many such arrangements are there that do not contain either *eye* or *ram*?
   Arrangements with neither eye nor ram $= \frac{7!}{2!2!} - \frac{5!}{2!} - \frac{5!}{2!} + 3!$

3. (10 points) Suppose you are interested in buying pizzas, and each pizza gets up to 7 toppings from 7 possible types (no double toppings).

(a) How many ways can you choose toppings for a pizza?
Each pizza can have 0 to 7 toppings. Therefore, there are 2^7 **different pizzas.**

(b) How many ways can you choose two pizzas with the same toppings?
It is the same as the number of different pizzas. The answer is 2^7**.**

(c) How many ways can you choose toppings for two pizzas?
Since we can have two pizzas with the same toppings, the problem is combination with repetitions. There are 2^7 **different pizzas, and we need to select two of them where repetitions are allowed. Therefore, the answer is** $\binom{2-1+2^7}{2^7-1} = \binom{2^7+1}{2}$

(d) How many ways can you choose toppings for n pizzas?
We now select n **pizzas from** 2^7 **different toppings ones. The answer is** $\binom{n-1+2^7}{2^7-1}$**.**

4. (10 points) We have seen that the following problem captures many counting problems.
Determine the number of non-negative integer solutions to

$$\begin{gathered} x_1 + x_2 + ... + x_k = n \\ x_i \geq 0, i = 1, 2, ..., k. \end{gathered}$$

Formulate each of the following problems as a variation of the above problem.

(a) Determine the number of ways to select k objects with replacements from a set of n objects.
Ans:

$$\begin{gathered} x_1 + x_2 + ... + x_n = k \\ x_i \geq 0, i = 1, 2, ..., n. \end{gathered}$$

(b) Determine the number of ways to place n nondistinguisable balls in k boxes.
Ans:

$$\begin{gathered} x_1 + x_2 + ... + x_k = n \\ x_i \geq 0, i = 1, 2, ..., k. \end{gathered}$$

(c) Determine the number of ways to distribute n pennies to k kids such that each kid gets at least 1 penny.
Ans:

$$x_1 + x_2 + ... + x_k = n$$
$$x_i \geq 1, i = 1, 2, ..., k.$$

(d) Determine the number of times the following pseudocode prints the PRINT statement:

```
for i = 1 to 20
   for j = 1 to i
      for k = 1 to j
         PRINT(i,j,k)
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Ans:

$$x_1 + x_2 + ... + x_{20} = 3$$
$$x_i \geq 0, i = 1, 2, ..., 20.$$

Once the three integers are selected, we assign the largest one to i, the smallest one to k and the third one to j.

5. (10 points) For each of the following deductive arguments, translate it into propositional logic notation using logical operators ($\neg, \vee, \wedge, \rightarrow, \leftrightarrow$), and then identify the premises (hypotheses) and conclusions, followed by a validity test.

(a) Either A or B is telling the truth. Either C or B is lying. Thus either A is telling the truth or C is lying.
Ans: It is valid
The arguments can be written as:

p: A is telling the truth.
q: B is telling the truth.
r: C is telling the truth.

1. $p \vee q$
2. $\neg r \vee \neg q$

Therefore, $p \vee \neg r$.

From (1), either p is true, or q is true. If q is true, $\neg r$ is true (from (2)). In this case the conclusion is true. If q is false, p is true. In this case also the conclusion is true.

(b) The main course will be chicken or fish. The vegetable will be carrots and broccoli. We will not have both fish as a main course and broccoli as a vegetable. Therefore, we will not have both chicken as a main course

and carrots as a vegetable.
Ans: It is not valid.
The arguments can be written as:

p: The main course will be chicken.
q: The main course will be fish.
r: Carrots will be served.
s: Broccoli will be served.

1. $p \vee q$
2. $r \wedge s$
3. $\neg(q \wedge s)$

Therefore, $\neg(p \wedge r)$

From (2), r and s are both true. Due to (3) and s is true, q is false. Therefore from (1), p is true, since q is false. This implies that $p \wedge r$ is true. Therefore $\neg(p \wedge r)$ is false.

6. (5 points) Let p, q be primitive statements for which the implication $p \to q$ is false. Determine the truth values for each of the following.
(a) $p \wedge q$, **(b)** $\neg p \vee q$, **(c)** $q \to p$ **(d)** $\neg q \to \neg p$.
Answer:
Since $p \to q$ is false, we know that p is true and q is false. Therefore,

- $p \wedge q$ is false.
- $\neg p \vee q$ is false.
- $q \to p$ is true.
- $\neg q \to \neg p$ is false.

7. (5 points) Determine a truth value assignment, if any, for the primitive statements p, q, r, s, t that make each of the following statements false. I have used $\oplus$ to indicate **exclusive-or** connective.

   (a) $[(p \wedge q) \wedge r] \to (s \vee t)$
   Answer:
   Clearly, we need to set the left side of the implication statement to false. This is achieved when p, q and r are set to true, and s and t are set to false.

   (b) $[p \wedge (q \wedge r)] \to (s \oplus t)$
   Answer:
   We can make the implication false by setting p, q and r true and setting

$s \oplus t$ false. We know that $s \oplus t$ is false when both s and t are true, or both s and t are false.

8. (10 points) We have seen in the class that any compound statement can be transformed to an equivalent statement involving only $\neg$, $\vee$ and $\wedge$ connectives. Show that $p \oplus q$ is equivalent to $(p \vee q) \wedge \neg(p \vee q)$. Proving equivalence through truth table is not acceptable.

   Answer:

   The answer to the question as presented is $p \oplus q$ cannot be equivalent to $(p \vee q) \wedge \neg(p \vee q)$. Clearly, $(p \vee q) \wedge \neg(p \vee q)$ is always false because it is of type $r \wedge \neg r$.

   However, as shown below, $p \oplus q$ is equivalent to $(p \vee q) \wedge \neg(p \wedge q)$.

   We know that $p \oplus q$ is true when p is true and q is false, or p is false and q is true. Using the techniques developed in the class, we know that $p \oplus q$ is equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$. We have shown in the class that $(p \wedge \neg q) \wedge (\neg p \wedge q)$ is then equivalent to $(p \vee q) \wedge \neg(p \wedge q)$.