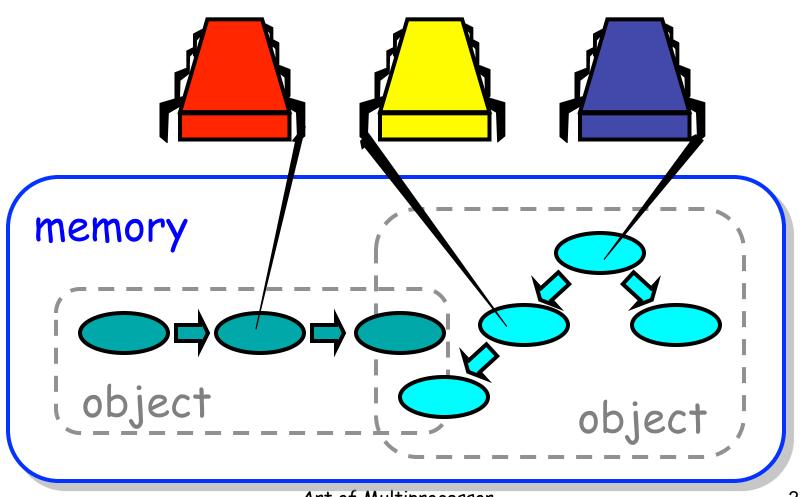


Companion slides for
The Art of Multiprocessor Programming
by Maurice Herlihy & Nir Shavit

### Concurrent Computation



### Objectivism

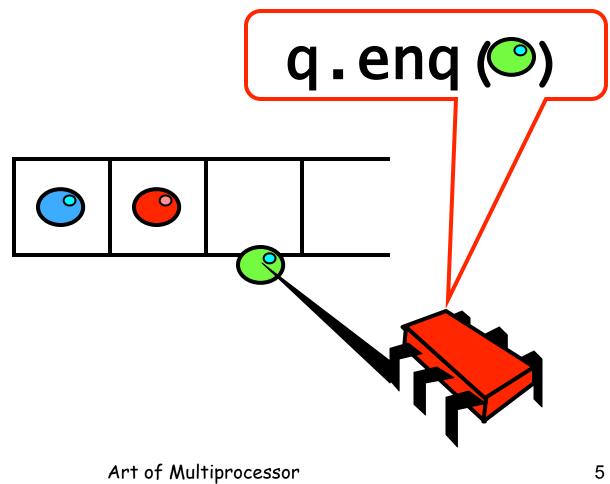
- What is a concurrent object?
  - How do we describe one?
  - How do we implement one?
  - How do we tell if we're right?

### Objectivism

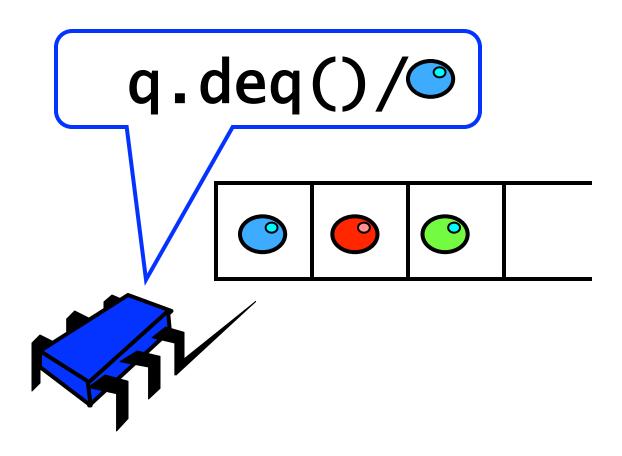
- What is a concurrent object?
  - How do we describe one?
  - How do we tell if we're right?

#### FIFO Queue: Enqueue Method

Programming



#### FIFO Queue: Dequeue Method



#### A Lock-Based Queue

```
class LockBasedQueue<T> {
  int head, tail;
  T[] items;
  Lock lock;
  public LockBasedQueue(int capacity) {
    head = 0; tail = 0;
    lock = new ReentrantLock();
    items = (T[]) new Object[capacity];
}
```

#### A Lock-Based Queue

```
class LockBasedOueue<T> {
   int head, tail;
   T[] items;
   Lock lock;
   public LockBasedQueue(int capacity) {
     head = 0, tail = 0;
     lock = new ReentrantLock();
     items = (T[]) new Object[capacity];
}
```

Queue fields protected by single shared lock

#### A Lock-Based Queue

```
head
                                          tail
                            capacity-1
class LockBasedQueue<T> {
  int head, tail;
  T[] items;
  Lock lock:
  public LockBasedQueue(int capacity) {
    head = 0; tail = 0;
    lock = new ReentrantLock();
    items = (T[]) new Object[capacity];
                    Initially head = tail
```

```
head
                                              tail
public T deq() throws EmptyExcention
  lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
```

```
head
                                             tail
               throws EmptyException
public T dea()
  lock.lock();
    if (tail = head)
       throw new EmptyException();
   T x = items[head % items.length];
    head++;
    return x;
  } finally {
                             Method calls
    lock.unlock();
                           mutually exclusive
```

```
head
                                             tail
public T deq() throws EmptyExcention
  lock.lock();
    if (tail == head)
       throw new EmptyException();
   T x = items[head \% items. ength];
    head++;
    return x;
  } finally {
                            If queue empty
    lock.unlock();
                            throw exception
```

```
head
                                            tail
public T deq() throws EmptyExcention
 lock.lock();
  try {
    if (tail == head)
       throw new EmptyException()
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
                           Queue not empty:
    lock.unlock();
                       remove item and update
                                  head
```

```
head
                                              tail
public T deq() throws EmptyExcention
 lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
    T x = items[head % items.length];
    head++:
    return x;
   finally {
                              Return result
    lock.unlock();
```

```
head
                                             tail
public T deq() throws EmptyExcention
 lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
   T x = items[head % items.length];
    head++;
                              Release lock no
    return x:
    finally {
                               matter what!
    lock.unlock();
```

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
                      should be correct because
  } finally {
                      modifications are mutually
    lock.unlock();
                  Art of Mexclusive...
```

# Now consider the following implementation

- The same thing without mutual exclusion
- · For simplicity, only two threads
  - One thread enq only
  - The other deq only

#### Wait-free 2-Thread Queue

```
public class WaitFreeQueue {
  int head = 0, tail = 0;
  items = (T[]) new Object[capacity];
  public void eng(Item x) {
   while (tail-head == capacity); // busy-wait
    items[tail % capacity] = x; tail++;
  public Item deq() {
     while (tail == head);  // busy-wait
     Item item = items[head % capacity]; head++;
     return item;
}}
```

#### Wait-free 2-Thread Queue

```
public class LockFreeQueue {
                                               tail
                                     head
                                  capacity-1
  int head = 0, tail = 0;
  items = (T[]) new Object[capacity]
  public void enq(Item x) {
    while (tail-head == capacity); // busy-wait
    items[tail % capacity] = x; tail++;
  public Item deq() {
     while (tail == head);  // busy-wait
     Item item = items[head % capacity]; head++;
     return item;
}}
```

#### Lock-free 2-Thread Queue

```
public class LockFreeQueue {
                                        head
                                                  tail
                                     capacity-
  int head = 0, tail = 0;
  items = (T[])new Object[capacity]
  public void eng(Item x) {
    items[tail % capacity] = x; tail++;
                      How do we define "correct
    returus; is up when modifications are not exclusive?
  public Itam ded
}}
                                                     20
                      Programming
```

# Defining concurrent queue implementations

- Need a way to specify a concurrent queue object
- Need a way to prove that an algorithm implements the object's specification
- · Lets talk about object specifications

. . .

### Correctness and Progress

- In a concurrent setting, we need to specify both the safety and the liveness properties of an object
- Need a way to define
  - when an implementation is correct
  - the conditions under which it guarantees progress

Lets begin with correctness

### Sequential Objects

- · Each object has a state
  - Usually given by a set of fields
  - Queue example: sequence of items
- Each object has a set of methods
  - Only way to manipulate state
  - Queue example: enq and deq methods

### Sequential Specifications

- If (precondition)
  - the object is in such-and-such a state
  - before you call the method,
- Then (postcondition)
  - the method will return a particular value
  - or throw a particular exception.
- and (postcondition, con't)
  - the object will be in some other state
  - when the method returns,

# Pre and PostConditions for Dequeue

- Precondition:
  - Queue is non-empty
- Postcondition:
  - Returns first item in queue
- Postcondition:
  - Removes first item in queue

# Pre and PostConditions for Dequeue

- Precondition:
  - Queue is empty
- Postcondition:
  - Throws Empty exception
- Postcondition:
  - Queue state unchanged

# Why Sequential Specifications Totally Rock

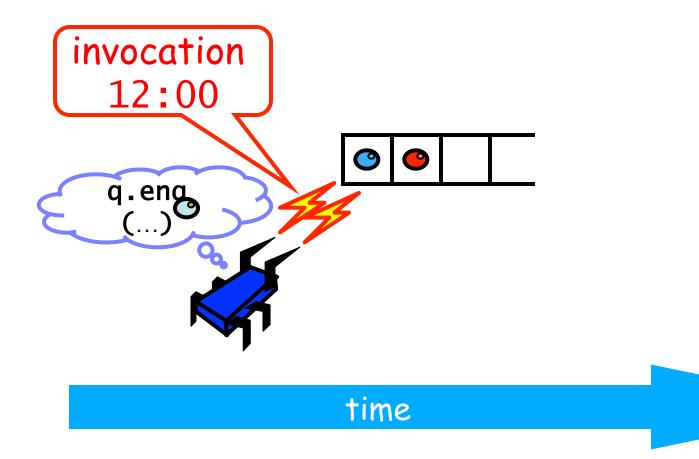
- Interactions among methods captured by side-effects on object state
  - State meaningful between method calls
- Documentation size linear in number of methods
  - Each method described in isolation
- Can add new methods
  - Without changing descriptions of old methods

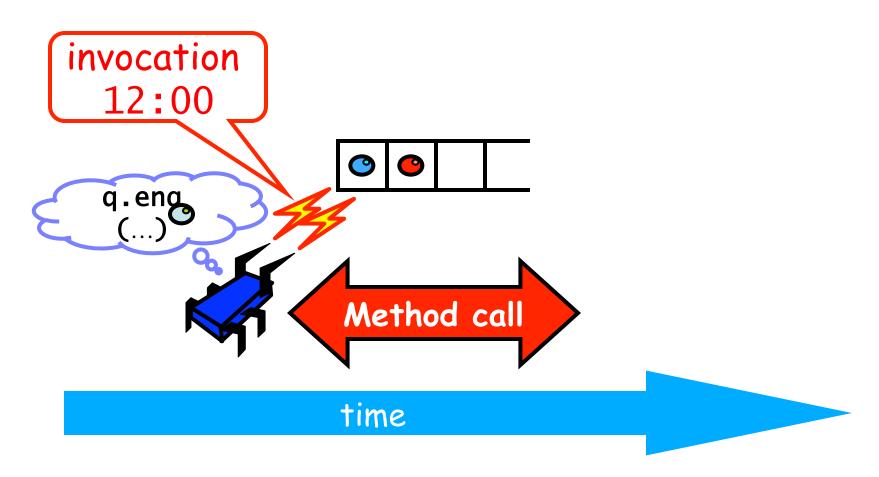
# What About Concurrent Specifications?

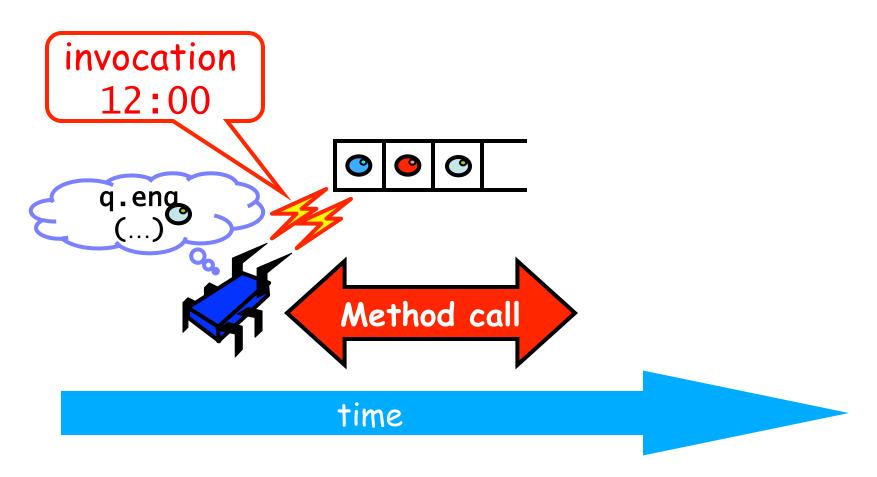
- Methods?
- Documentation?
- Adding new methods?

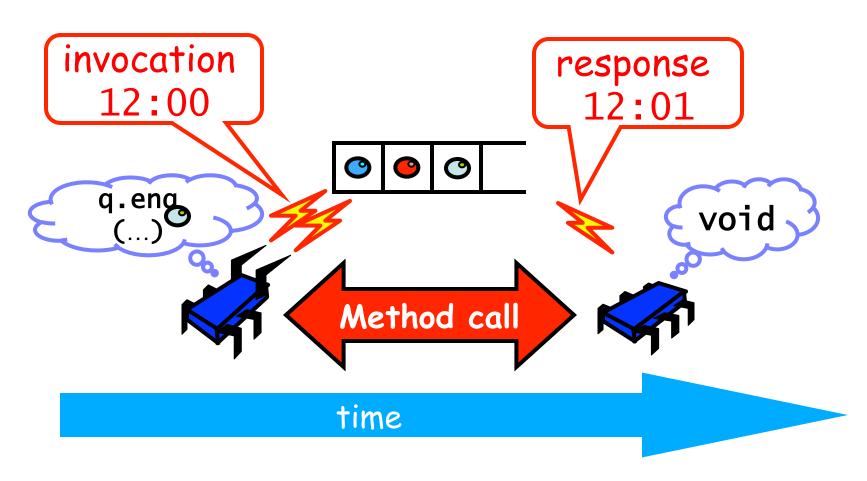


#### time









### Sequential vs Concurrent

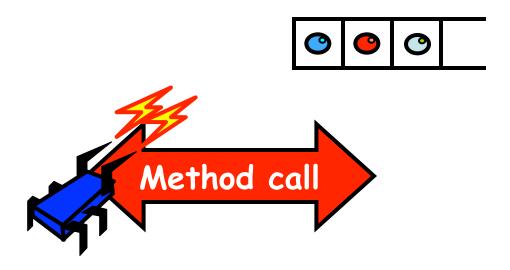
- Sequential
  - Methods take time? Who knew?
- Concurrent
  - Method call is not an event
  - Method call is an interval.

# Concurrent Methods Take Overlapping Time



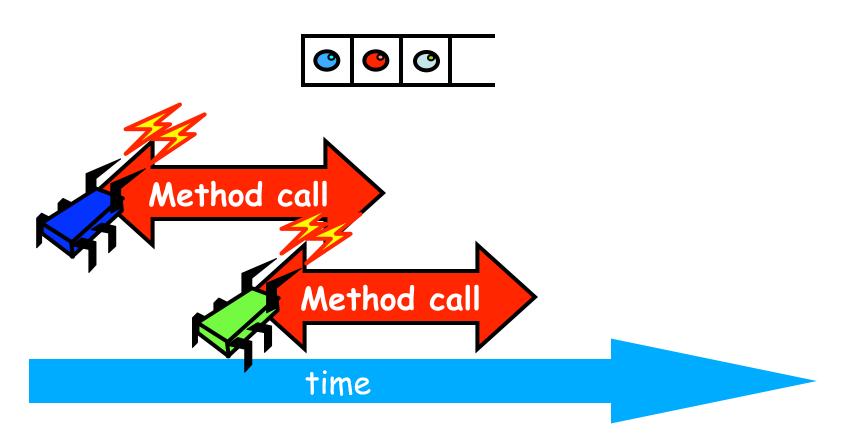
#### time

# Concurrent Methods Take Overlapping Time

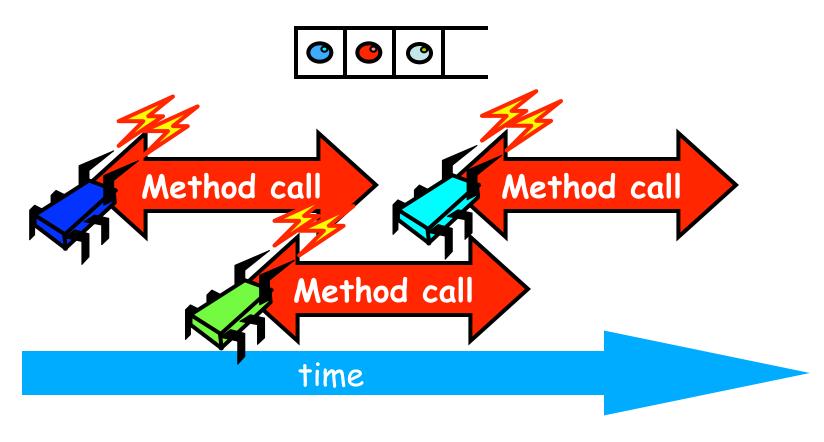


#### time

### Concurrent Methods Take Overlapping Time



# Concurrent Methods Take Overlapping Time



- Sequential:
  - Object needs meaningful state only between method calls
- Concurrent
  - Because method calls overlap, object might never be between method calls

- Sequential:
  - Each method described in isolation
- Concurrent
  - Must characterize *all* possible interactions with concurrent calls
    - · What if two engs overlap?
    - Two deqs? enq and deq? ....

#### Sequential:

 Can add new methods without affecting older methods

#### Concurrent:

- Everything can potentially interact with everything else

#### Sequential:

 Can add new methods without affecting older methods

#### · Concurrent:

- Everything can potentially interact with everything else

### The Big Question

- What does it mean for a concurrent object to be correct?
  - What is a concurrent FIFO queue?
  - FIFO means strict temporal order
  - Concurrent means ambiguous temporal order

### Intuitively...

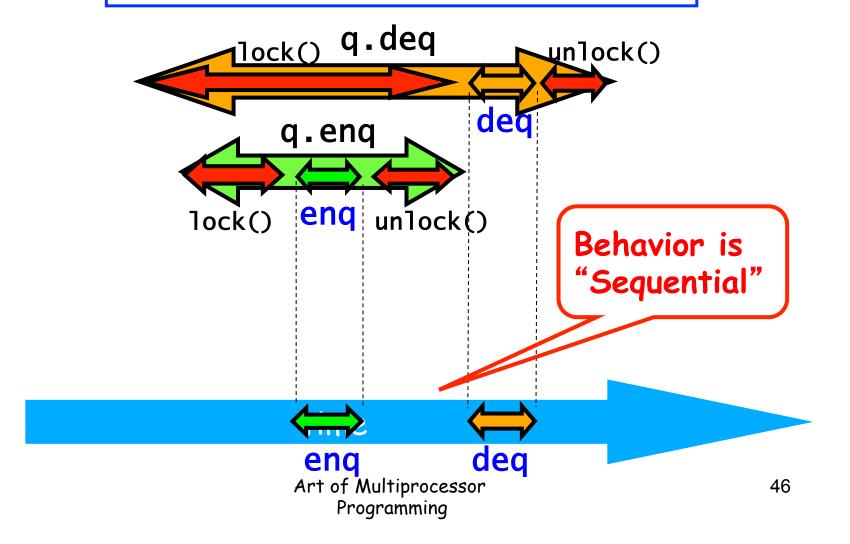
```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
  }
}
```

### Intuitively...

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail = head)
        throw New EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
    of queue are done
    mutually exclusive
```

#### **T.** \_ \_ . . . . \_ l.

#### Lets capture the idea of describing the concurrent via the sequential

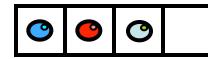


### Linearizability

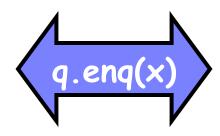
- Each method should
  - "take effect"
  - Instantaneously
  - Between invocation and response events
- Object is correct if this "sequential" behavior is correct
- Ordering must be maintained between request and responses (addendum)
- · Any such concurrent object is
  - Linearizable™

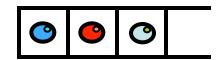
### Is it really about the object?

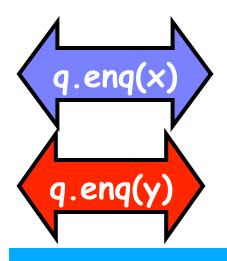
- Each method should
  - "take effect"
  - Instantaneously
  - Between invocation and response events
- Sounds like a property of an execution...
- A linearizable object: one all of whose possible executions are linearizable

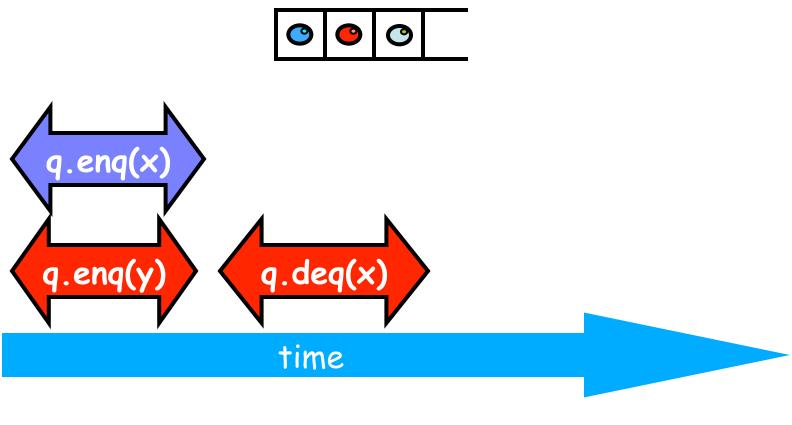




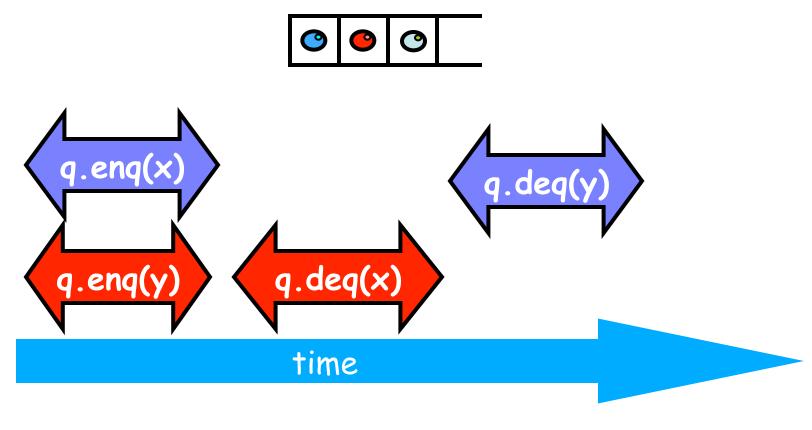


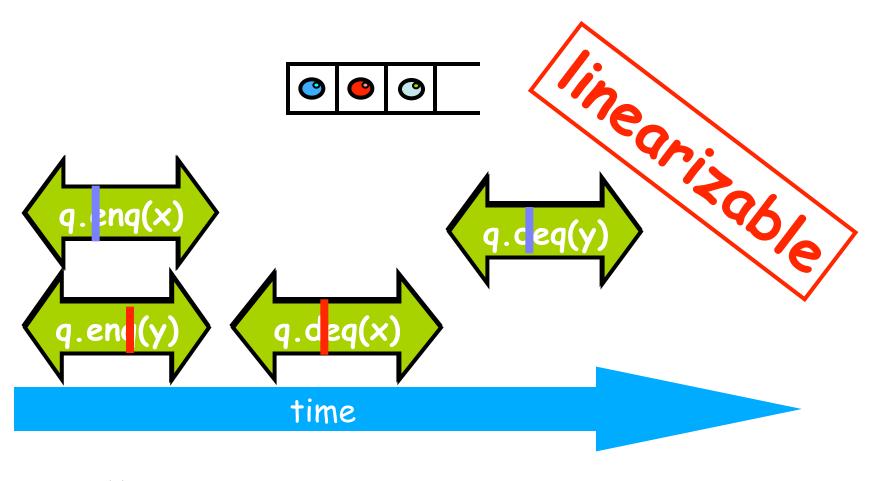


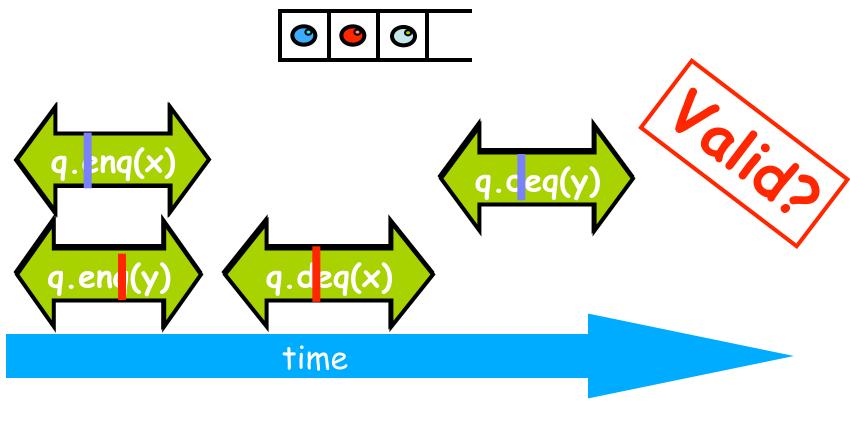


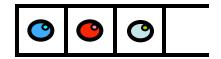


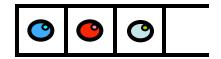


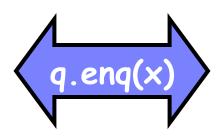


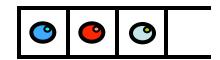


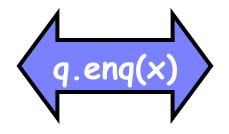


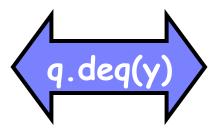




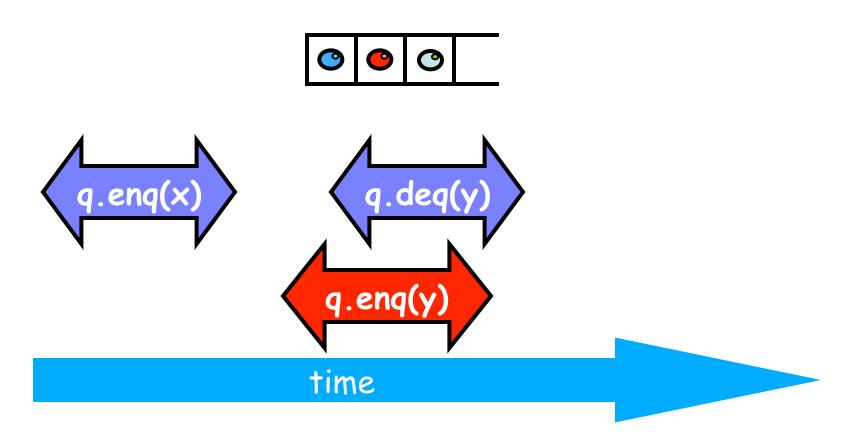




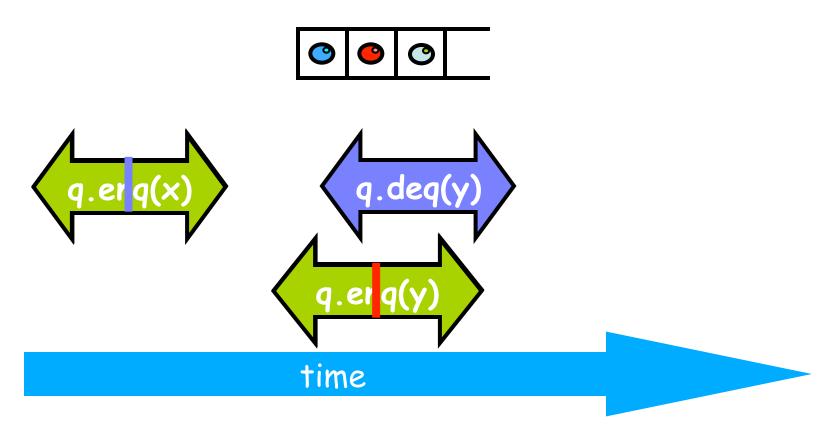


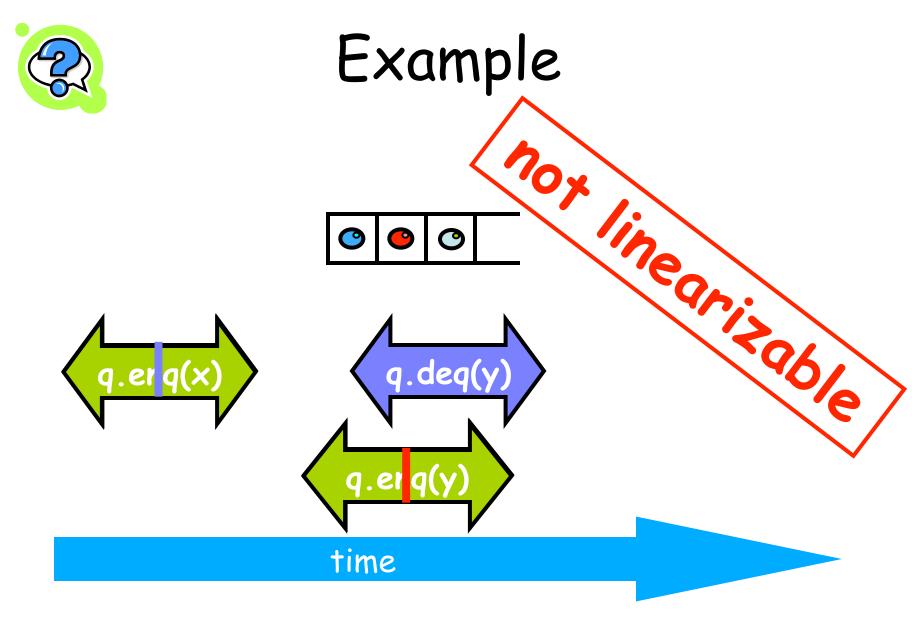


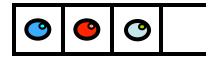


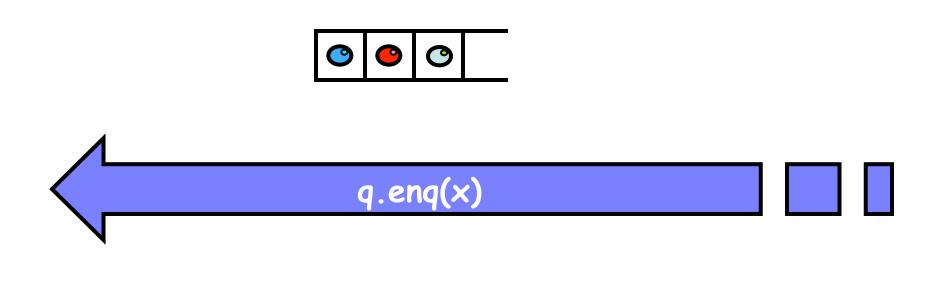




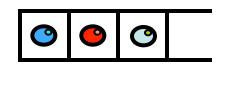


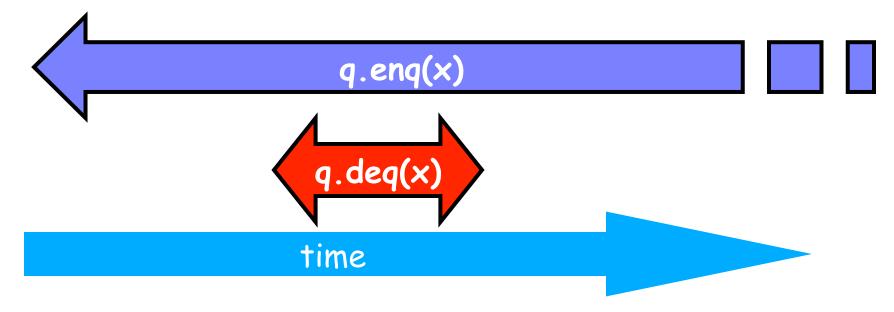




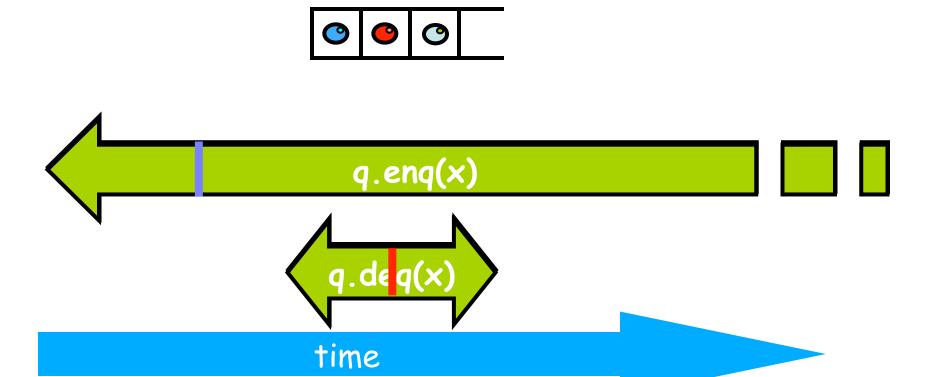




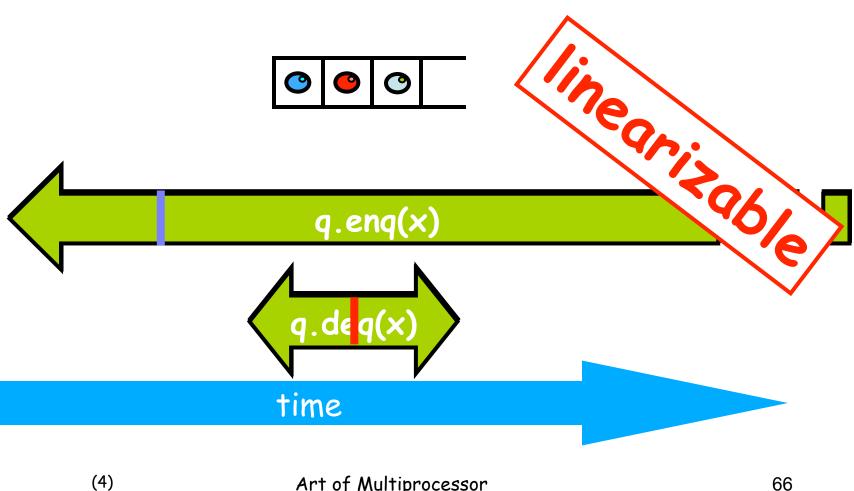




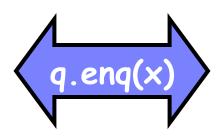


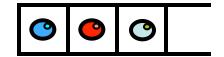


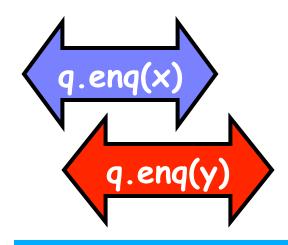


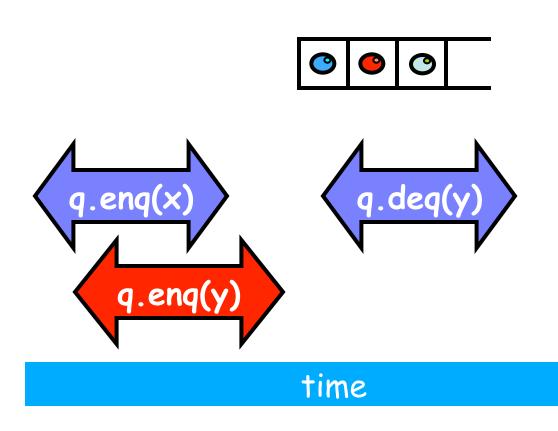




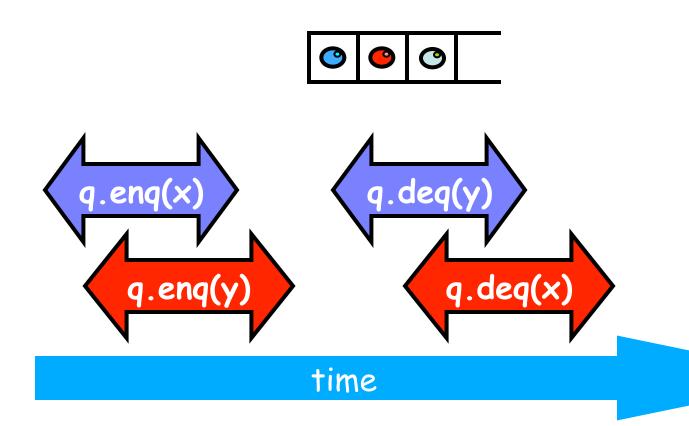


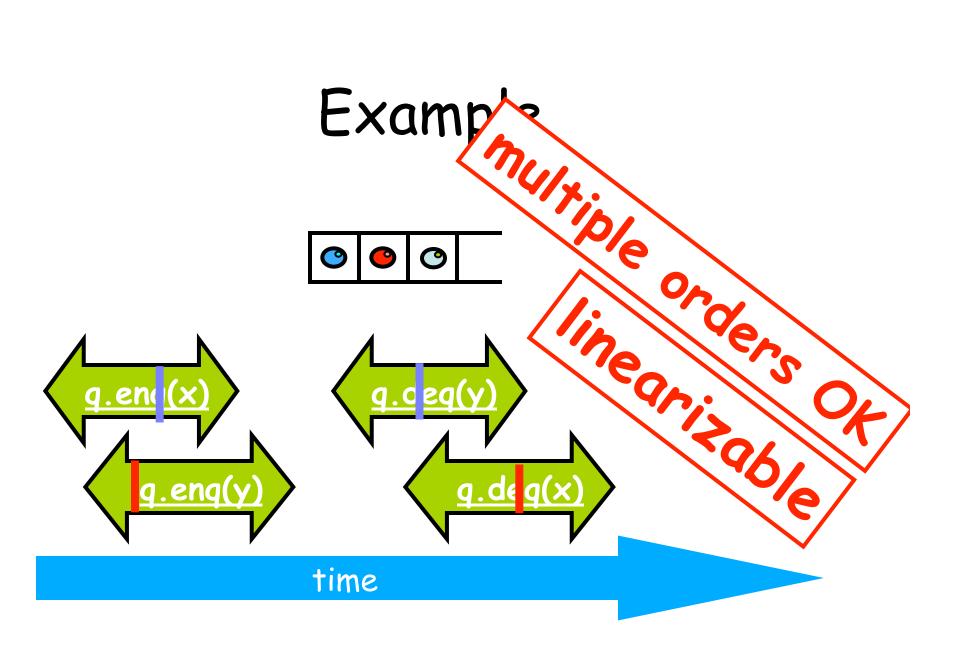




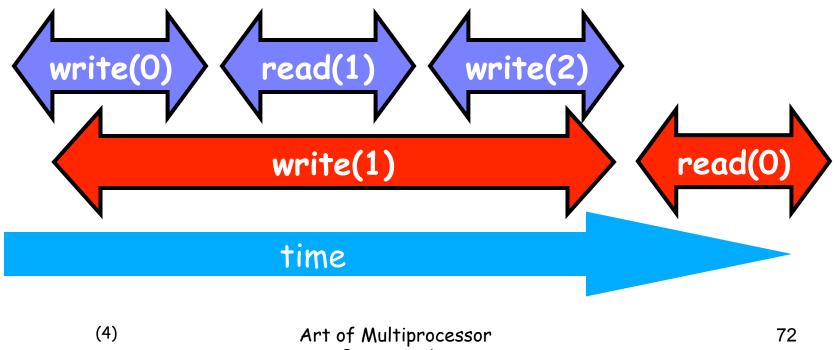


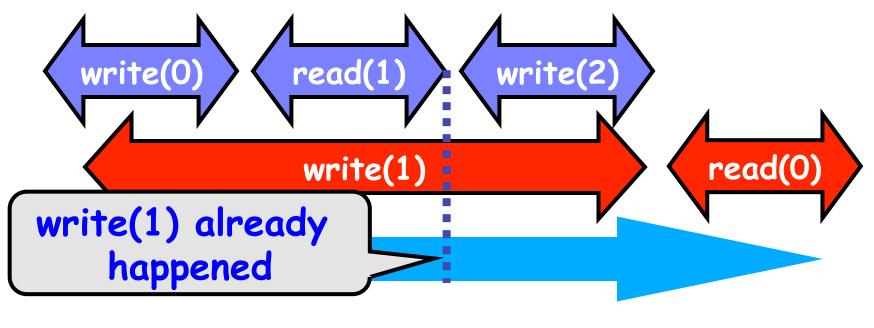


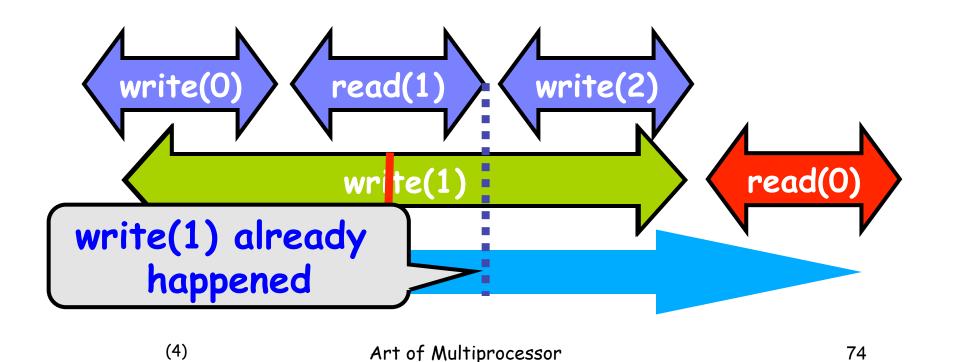




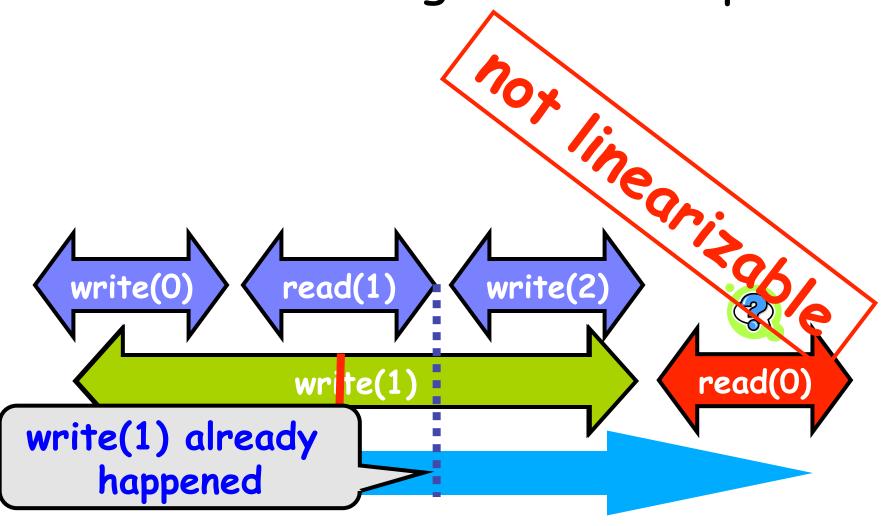
#### Read/Write Register Example

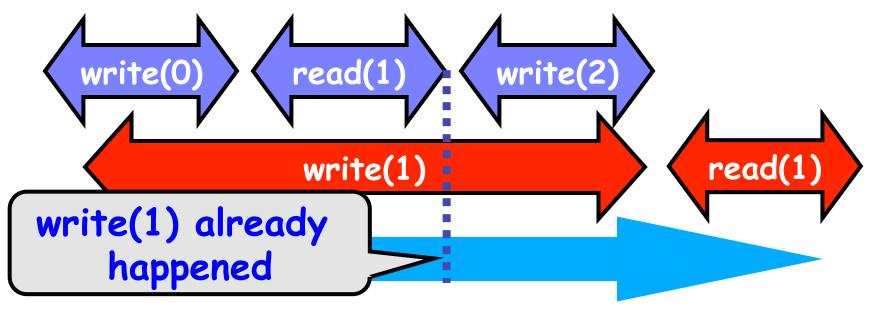


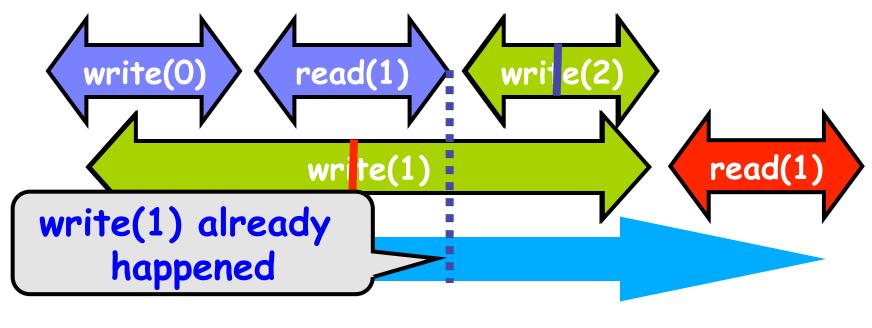


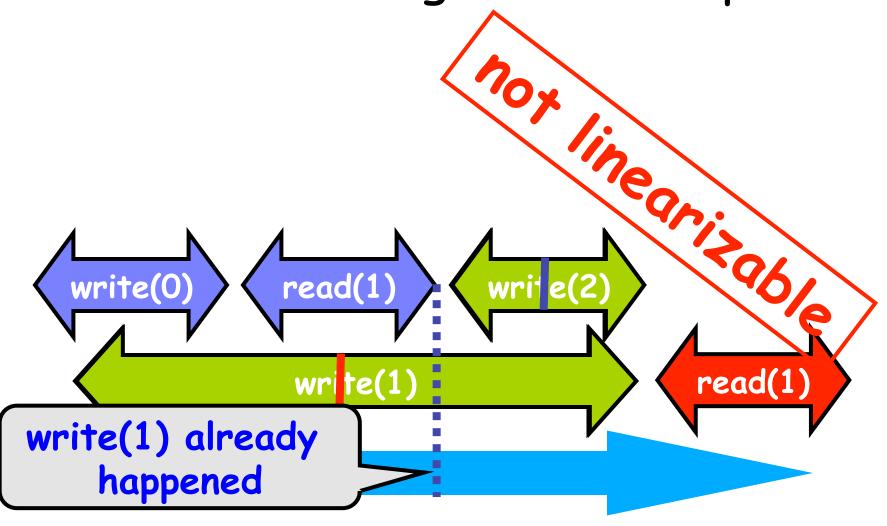


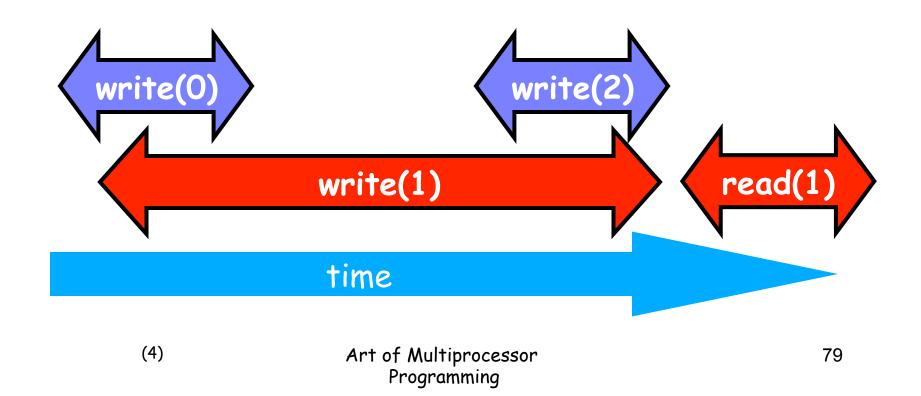
Programming

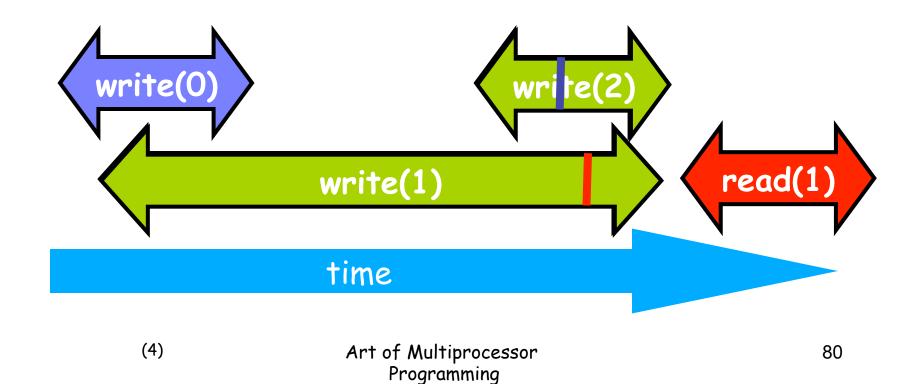


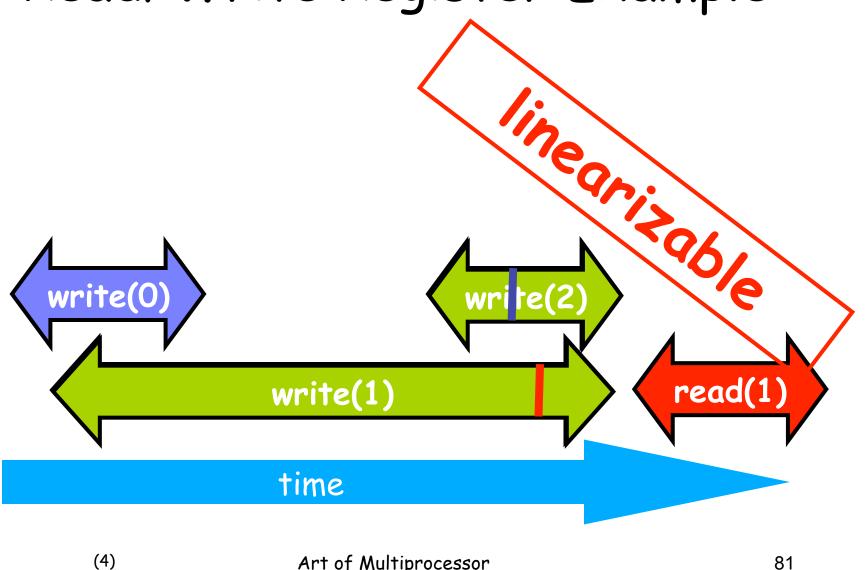


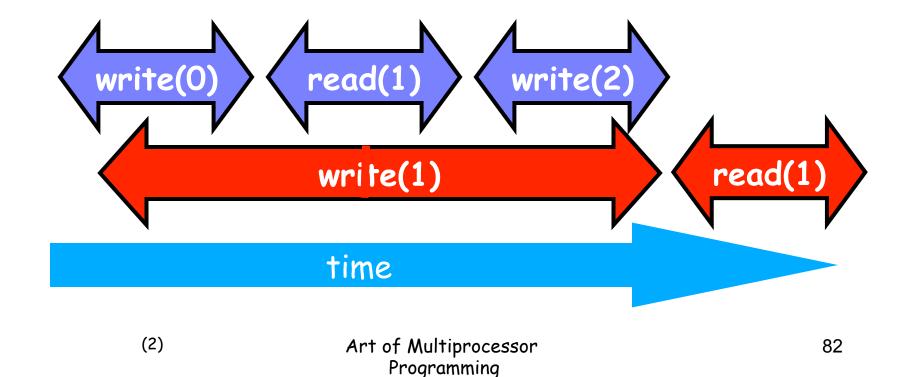


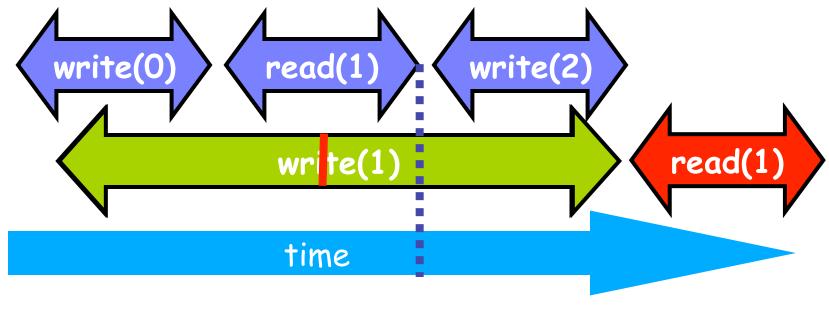


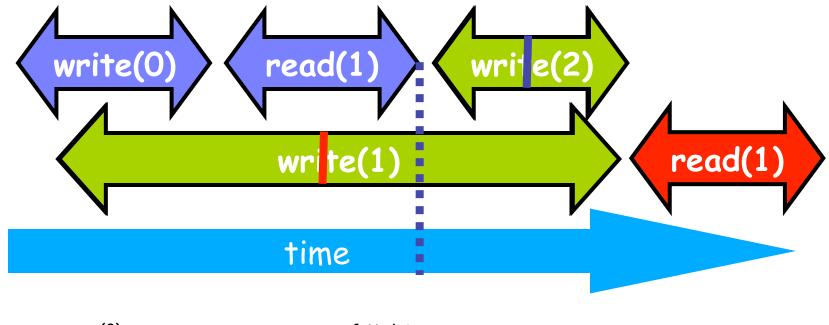


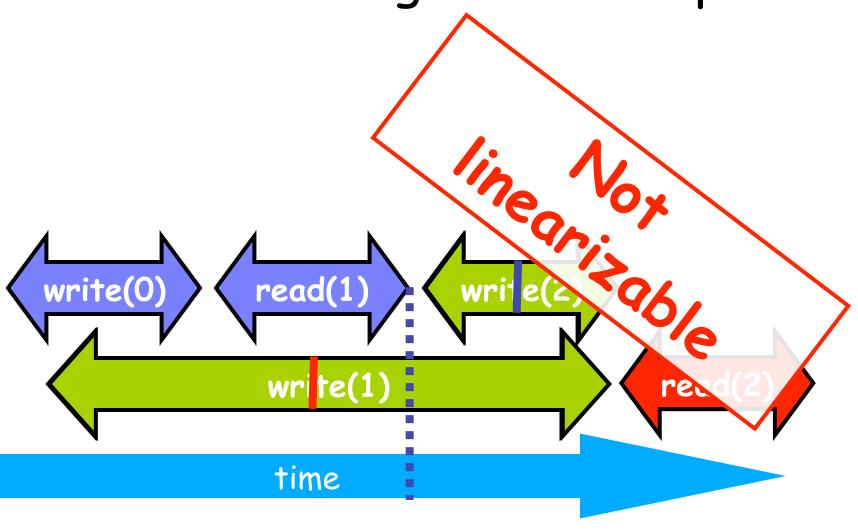












# Talking About Executions

- Why?
  - Can't we specify the linearization point of each operation without describing an execution?
- Not Always
  - In some cases, linearization point depends on the execution

#### Formal Model of Executions

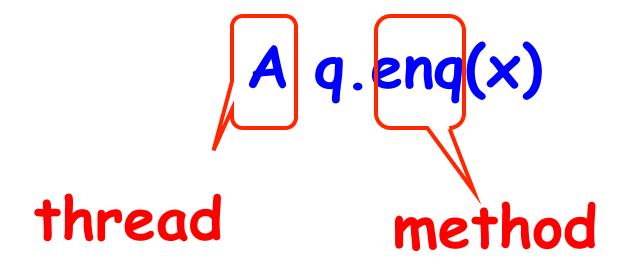
- Define precisely what we mean
  - Ambiguity is bad when intuition is weak
- Allow reasoning

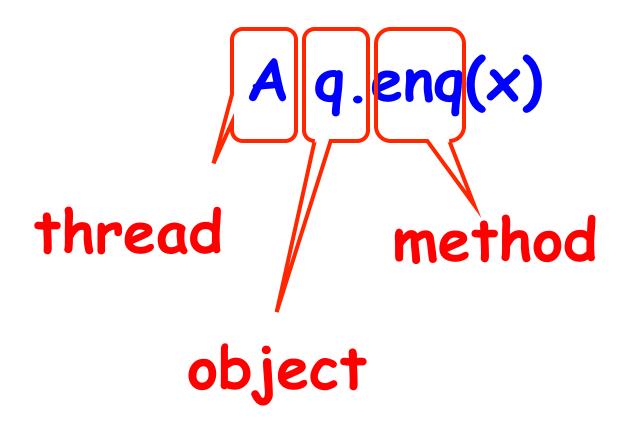
# Split Method Calls into Two Events

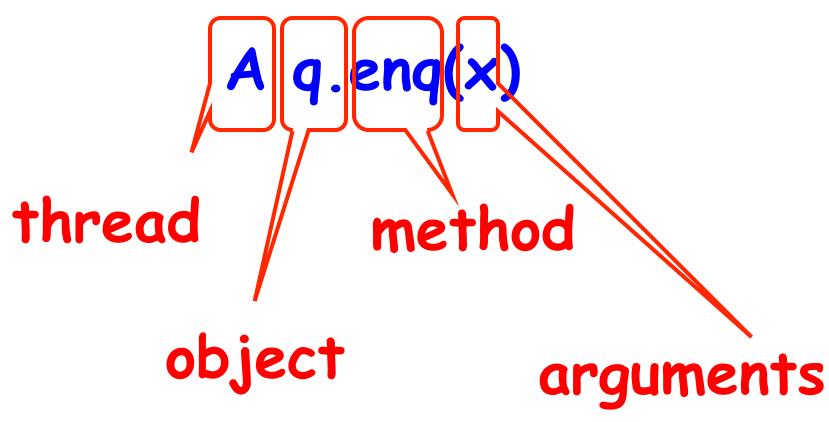
- Invocation
  - method name & args
  - -q.enq(x)
- Response
  - result or exception
  - -q.enq(x) returns void
  - -q.deq() returns x
  - -q.deq() throws empty

A q.enq(x)

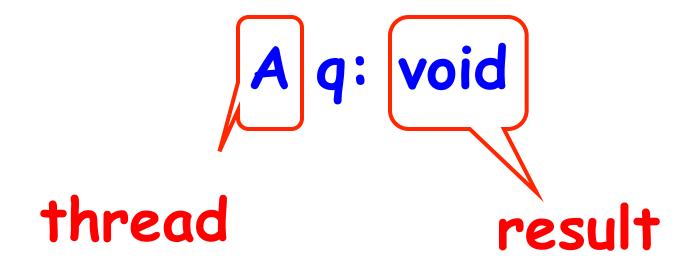
thread

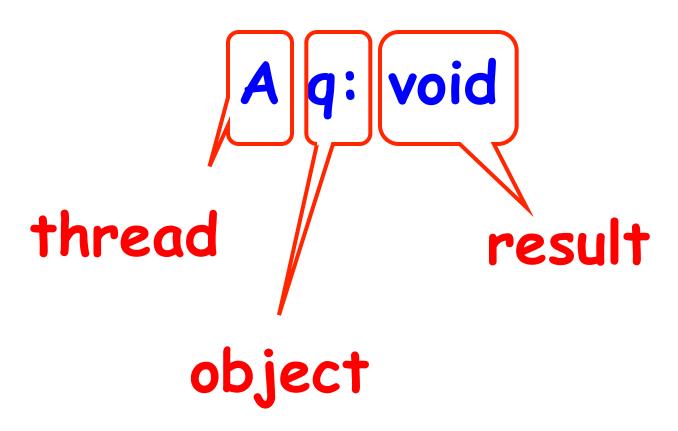


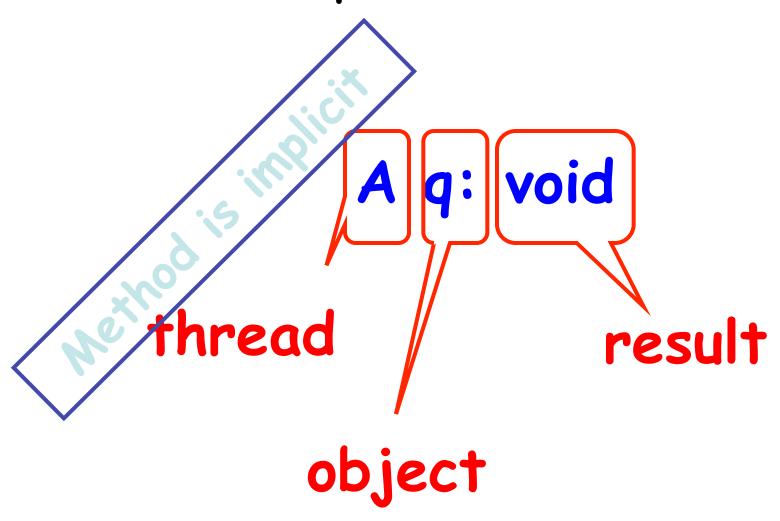


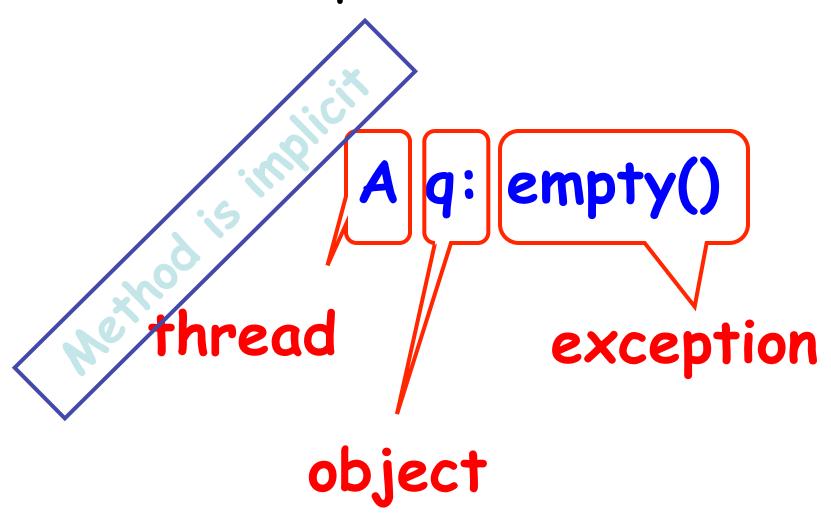


A q: void









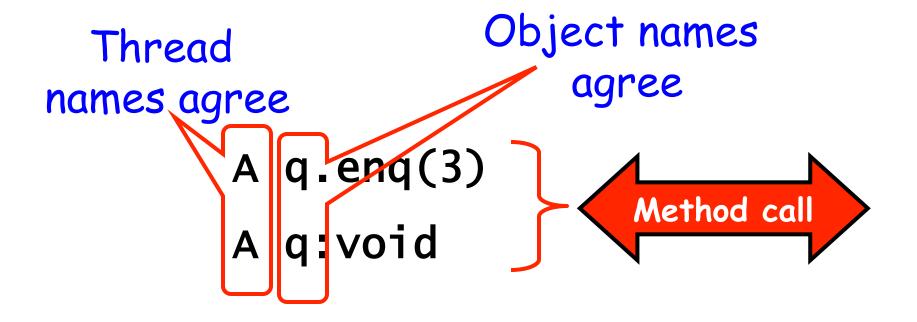
#### History - Describing an Execution

```
A q.enq(3)
A q:void
A q.enq(5)
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Sequence of invocations and responses

#### Definition

Invocation & response match if



# Object Projections

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

# Object Projections

```
A q.enq(3)
A q:void
```

$$H|q =$$

B q.deq()

B q:3

# Thread Projections

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

# Thread Projections

```
H|B = B p.enq(4)
B p:void
B q.deq()
B q:3
```

# Complete Subhistory

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq() An invocation is pending if it has no matching respnse
```

# Complete Subhistory

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq() May or may not
B q:3 have taken effect
```

# Complete Subhistory

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq() discard pending
B q:3 invocations
```

# Complete Subhistory

```
A q.enq(3)
A q:void
```

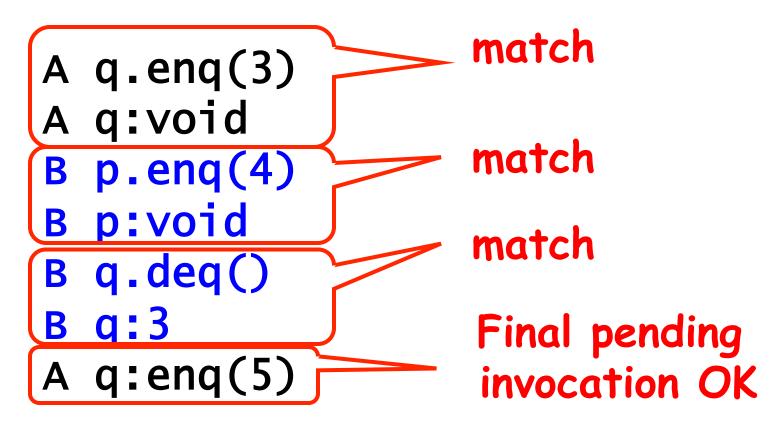
```
Complete(H) = B p.enq(4)
B p:void
B q.deq()
B q:3
```

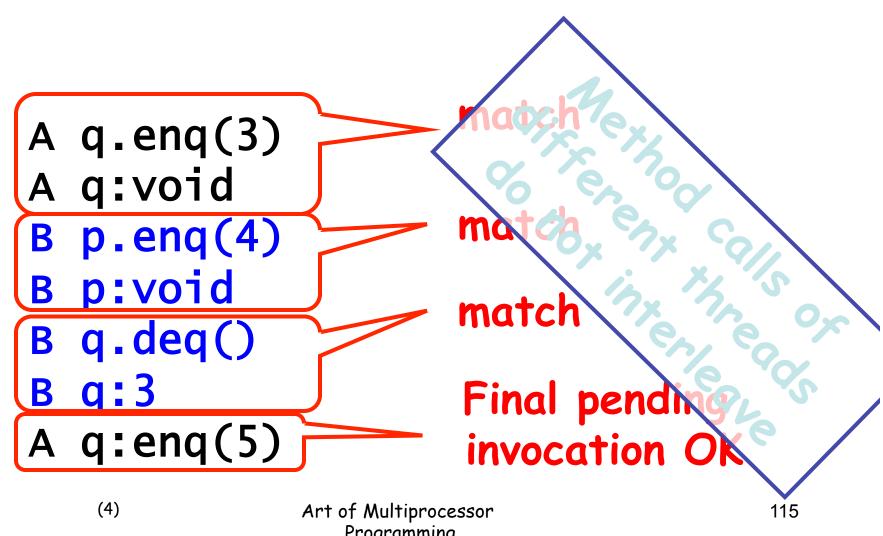
```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```





#### Well-Formed Histories

```
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

#### Well-Formed Histories

```
Per-thread
projections sequential

A q.enq(3)
B p.enq(4)
B p.void
B p.enq(4)
B p:void
H= B q.deq()
A q:void
B q:3
```

#### Well-Formed Histories

```
Per-thread

projections sequential

A q.enq(3)

B p.enq(4)

B p.enq(4)

B p.enq(4)

B p:void

B p:void

H= B q.deq()

A q:void

B q:3

H|A= A q.enq(3)

A q:void
```

### Equivalent Histories

```
Threads see the same  \begin{cases} H \mid A = G \mid A \\ H \mid B = G \mid B \end{cases}  thing in both  \begin{cases} H \mid B = G \mid B \end{cases}
```

```
H=
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

```
G= A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

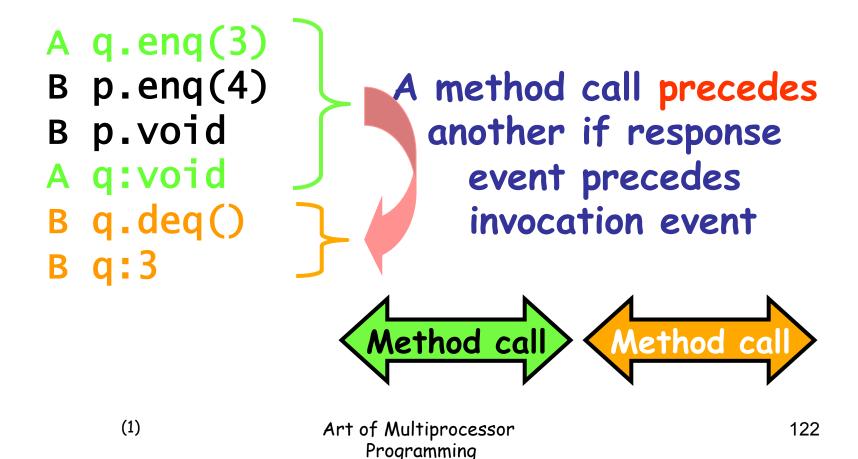
### Sequential Specifications

- A sequential specification is some way of telling whether a
  - Single-thread, single-object history
  - Is legal
- For example:
  - Pre and post-conditions
  - But plenty of other techniques exist ...

# Legal Histories

- A sequential (multi-object) history H is legal if
  - For every object x
  - H|x is in the sequential spec for x

#### Precedence



#### Non-Precedence

A q.enq(3)
B p.enq(4)
B p.void
B q.deq()
A q:void
B q:3

Method call
Method call

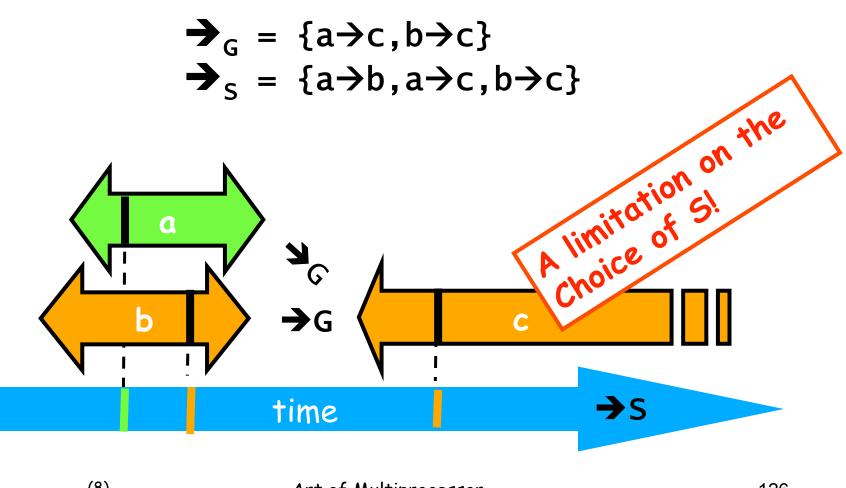
#### Notation

- · Given
  - History H
  - method executions  $m_0$  and  $m_1$  in H
- We say  $m_0 \rightarrow_H m_1$ , if
  - mo precedes m<sub>1</sub>
- Relation  $m_0 \rightarrow_H m_1$  is  $m_0 \rightarrow m_1$ 
  - Partial order
  - Total order if H is sequential

# Linearizability

- History H is *linearizable* if it can be extended to G by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
- So that G is equivalent to
  - Legal sequential history S
  - where  $\rightarrow_{G} \subset \rightarrow_{S}$

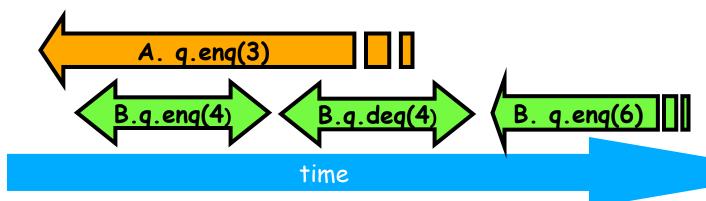
### What is $\rightarrow_G \subset \rightarrow_S$

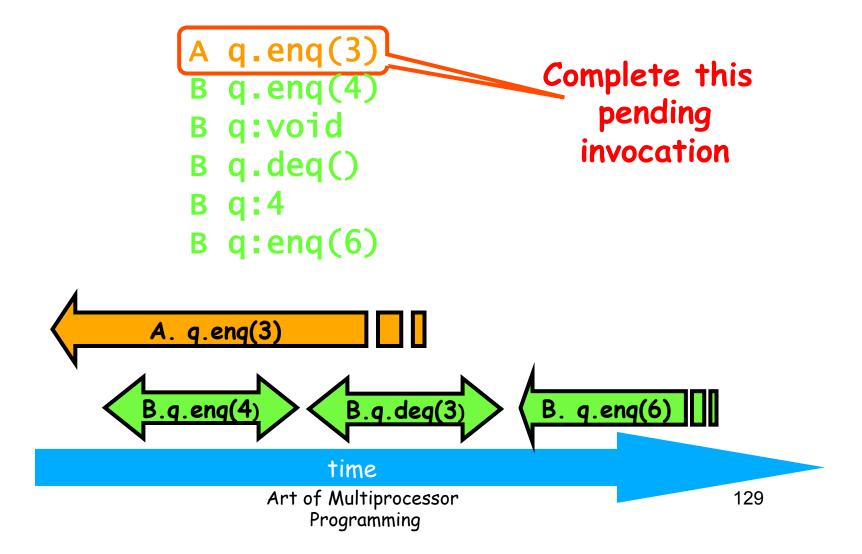


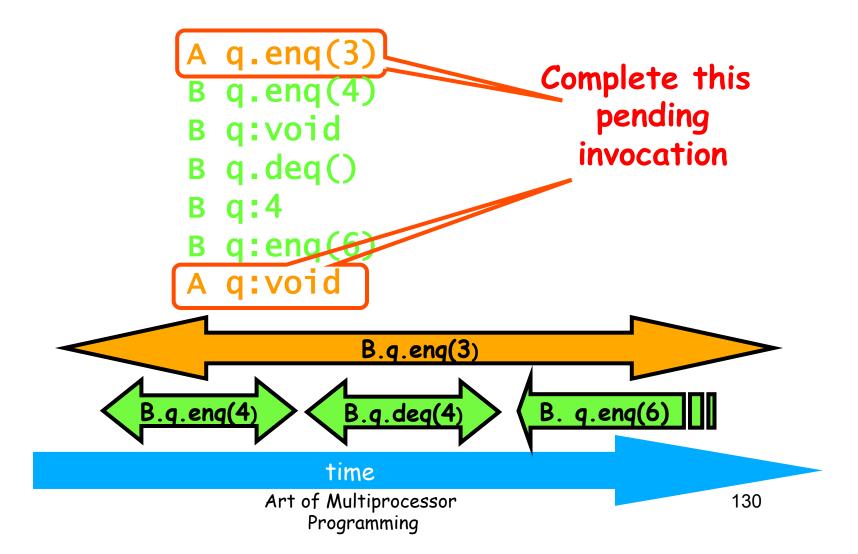
#### Remarks

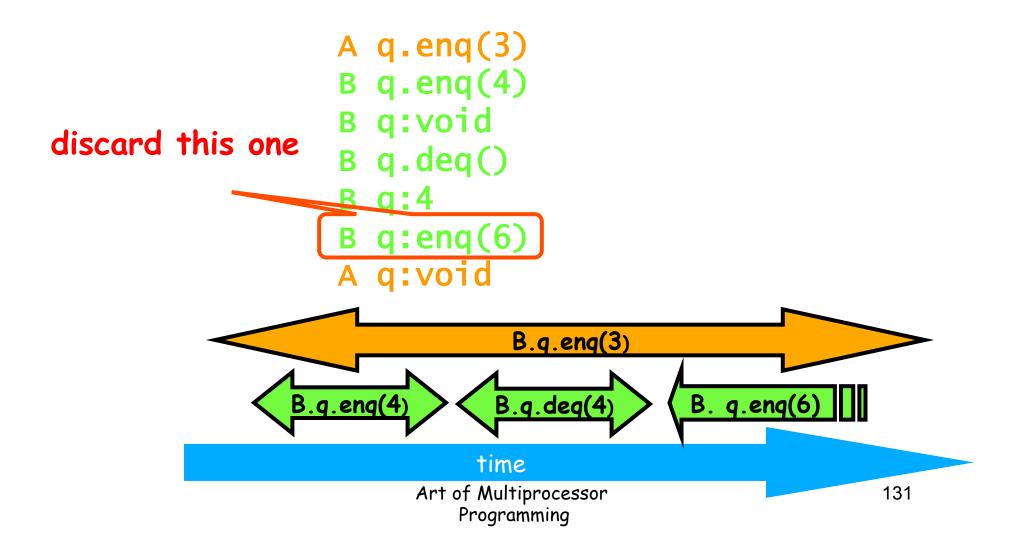
- Some pending invocations
  - Took effect, so keep them
  - Discard the rest
- Condition  $\rightarrow_{G} \subset \rightarrow_{S}$ 
  - Means that S respects "real-time order" of G

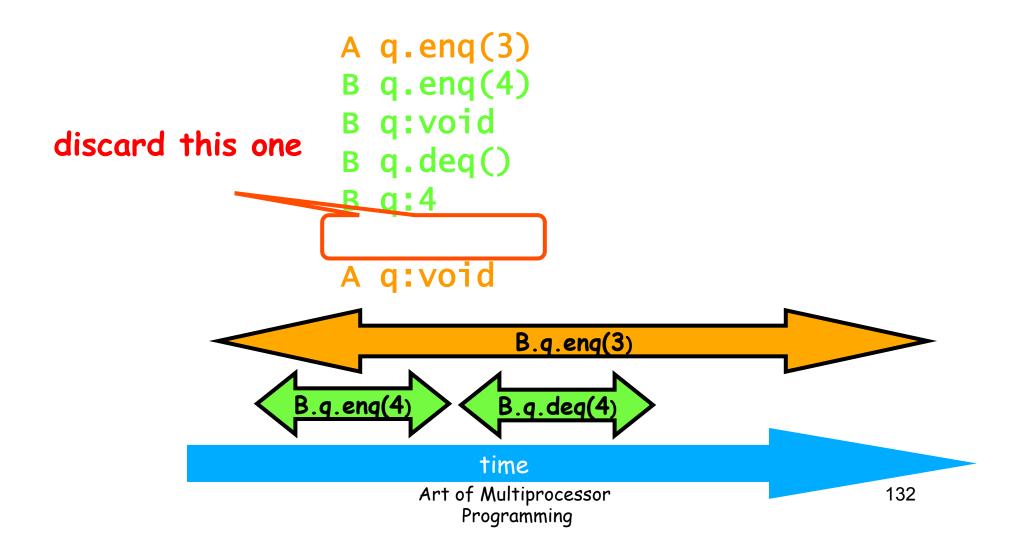
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```



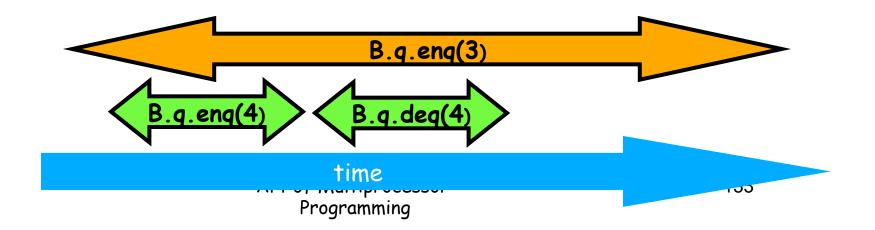








```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```



```
A q.enq(3)

B q.enq(4)

B q.enq(4)

B q:void

A q.enq(3)

B q.deq()

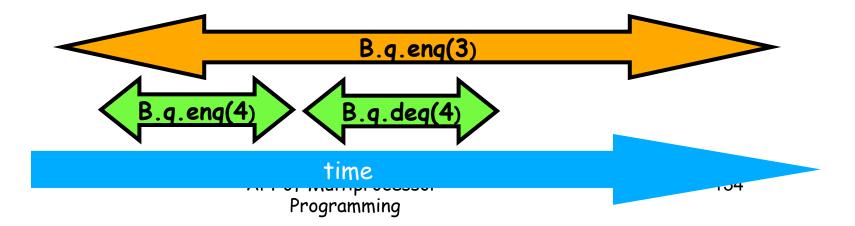
A q:void

B q:4

A q:void

B q:4

A q:void
```



#### Equivalent sequential history

```
Bq.enq(4)
A q.enq(3)
                          B q:void
B q.enq(4)
 q:void
                          A q.enq(3)
 q.deq()
                          A q:void
                          B q.deq()
 q:4
A q:void
                          B q:4
                     B.q.enq(3)
                    B.q. deq(4)
       B.q.erq(4)
                  time
                Programming
```

### Concurrency

- How much concurrency does linearizability allow?
- When must a method invocation block?

### Concurrency

- Focus on total methods
  - Defined in every state
- Example:
  - deq() that throws Empty exception
  - Versus deq() that waits ...
- · Why?
  - Otherwise, blocking unrelated to synchronization

### Concurrency

- Question: When does linearizability require a method invocation to block?
- · Answer: never.
- Linearizability is non-blocking

# Non-Blocking Theorem

```
If method invocation
  A q.inv(...)
is pending in history H, then there
 exists a response
  A q:res(...)
such that
  H + A q:res(...)
is linearizable
```

#### Proof

- Pick linearization S of H
- If S already contains
  - Invocation A q.inv(...) and response,
  - Then we are done.
- · Otherwise, pick a response such that
  - S + A q.inv(...) + A q:res(...)
  - Possible because object is total.

# Composability Theorem

- · History H is linearizable if and only if
  - For every object x
  - H|x is linearizable

#### Why Does Composability Matter?

- Modularity
- Can prove linearizability of objects in isolation
- Can compose independently-implemented objects

# Reasoning About Lineraizability: Locking

```
public T deq() throws EmptyException
lock.lock();
try {
   if (tail == head)
        throw new EmptyException();
   T x = items[head % items.length];
   head++;
   return x;
} finally {
   lock.unlock();
}
```

# Reasoning About Lineraizability: Locking

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  }
  finally {
  lock.unlock();
  }
  Linearization points
  are when locks are
  released
```

### More Reasoning: Lock-free

```
public class LockFreeQueue {
                                     head
                                               tail
                                  capacity-1
  int head = 0, tail = 0;
  items = (T[]) new Object[capacity]
  public void eng(Item x) {
    while (tail-head == capacity); // busy-wait
    items[tail % capacity] = x; tail++;
  public Item deq() {
     while (tail == head);  // busy-wait
     Item item = items[head % capacity]; head++;
     return item;
}}
```

### More Reasoning

```
public Item deq() {
     while (tail == head);  // busy-wait
     Item item = items[head % capacity]; head++;
     return item;
  }}
```

### Strategy

- Identify one atomic step where method "happens"
  - Critical section
  - Machine instruction
- Doesn't always work
  - Might need to define several different steps for a given method

### Linearizability: Summary

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being "atomic"
- There is a lot of ongoing research in verification community to build tools that can verify/debug concurrent implementations wrt linearizability

### Alternative: Sequential Consistency

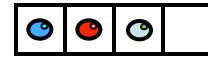
- · History H is Sequentially Consistent if it can be extended to G by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
- · So that G is equivalent to a Differs from linearizability
  - Legal sequential history S

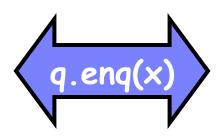


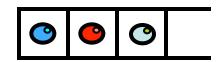
# Alternative: Sequential Consistency

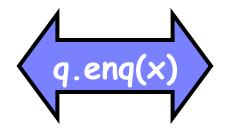
- No need to preserve real-time order
  - Cannot re-order operations done by the same thread
  - Can re-order non-overlapping operations done by different threads
- Often used to describe multiprocessor memory architectures

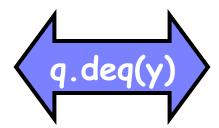




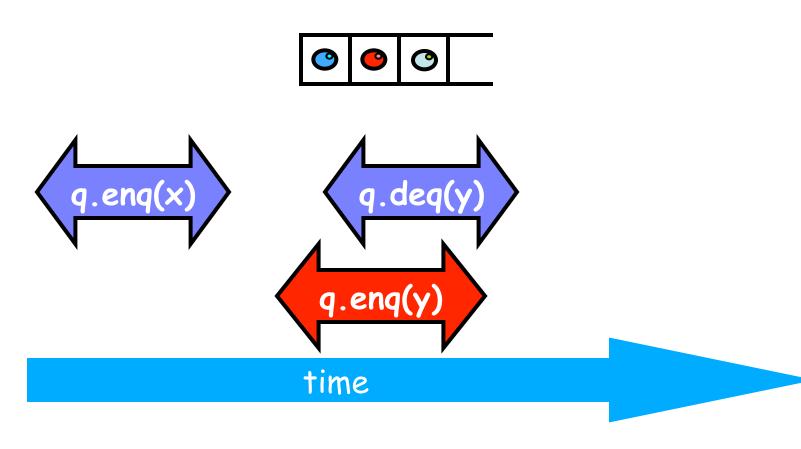




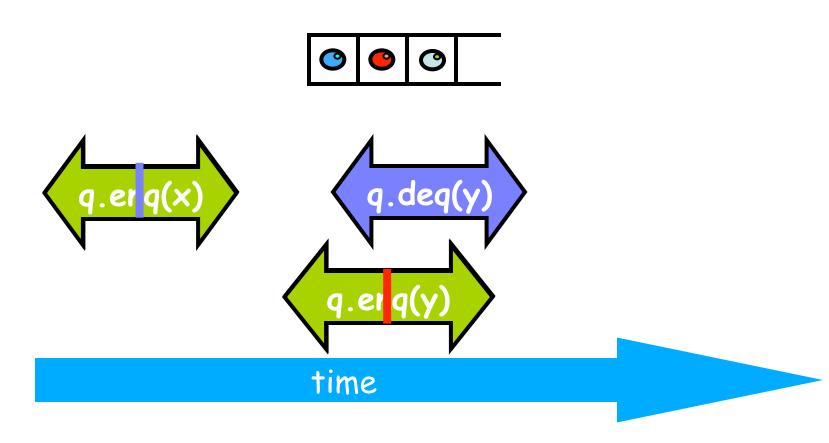


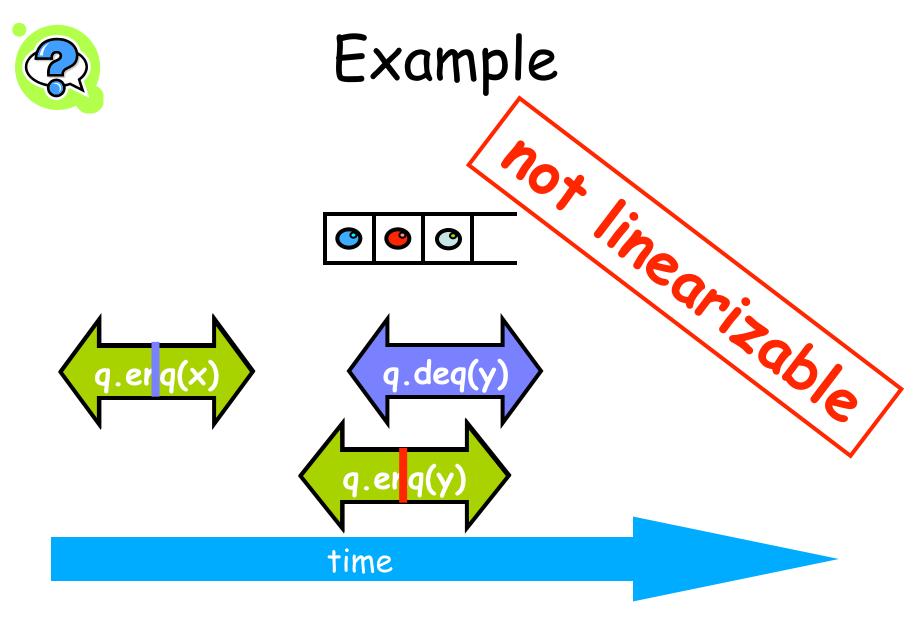


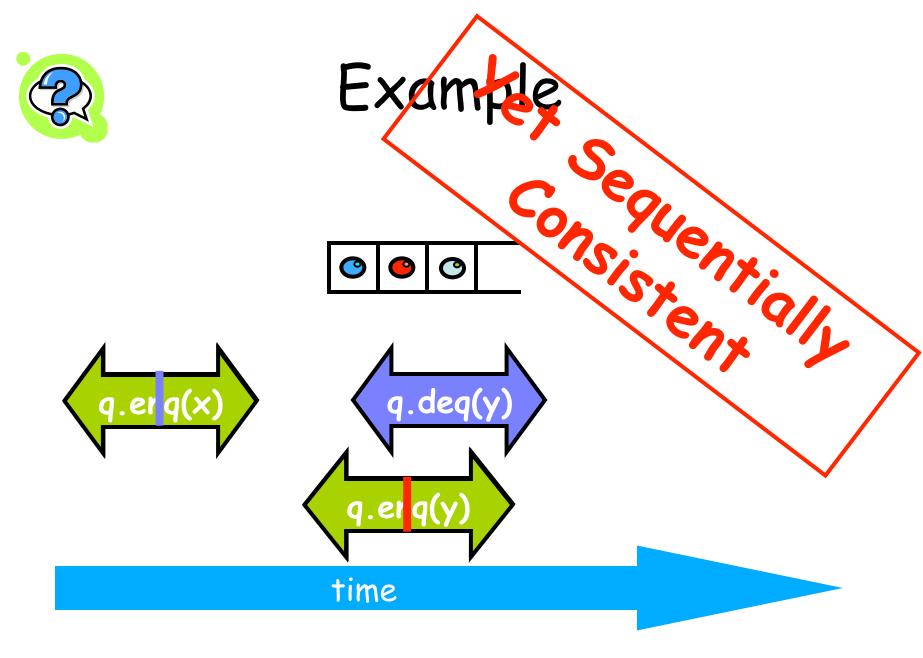










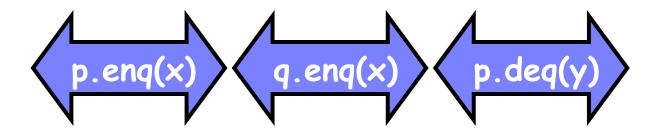


#### Theorem

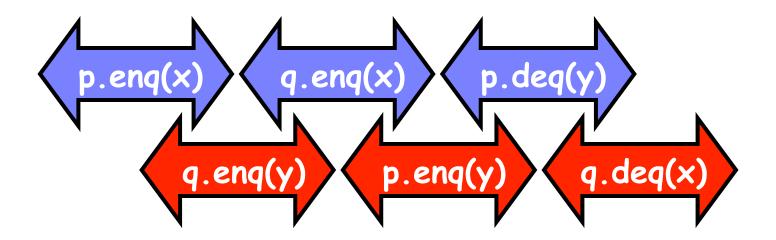
Sequential Consistency is not a local property

(and thus we lose composability...)

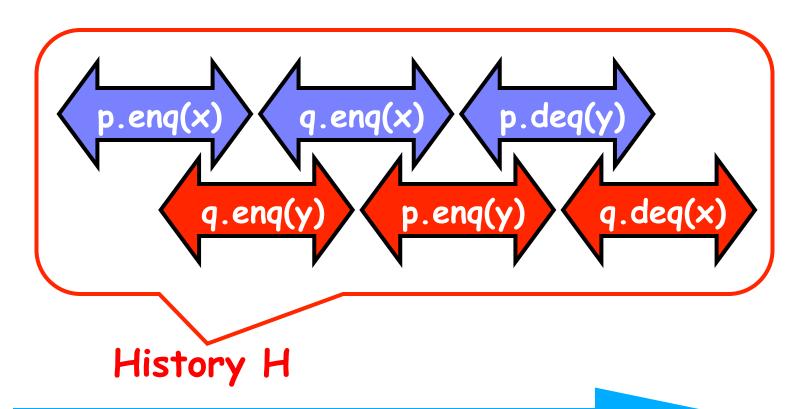
### FIFO Queue Example



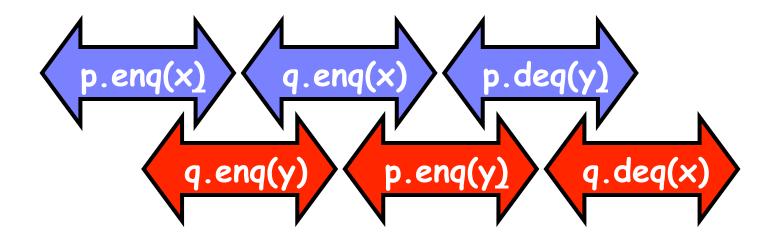
### FIFO Queue Example



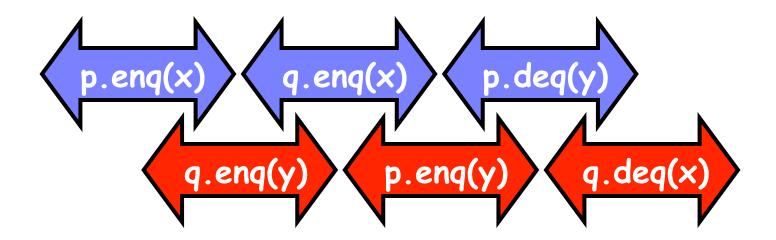
### FIFO Queue Example



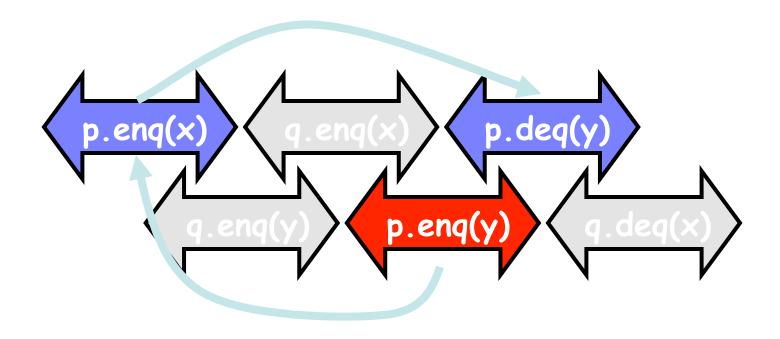
### H|p Sequentially Consistent



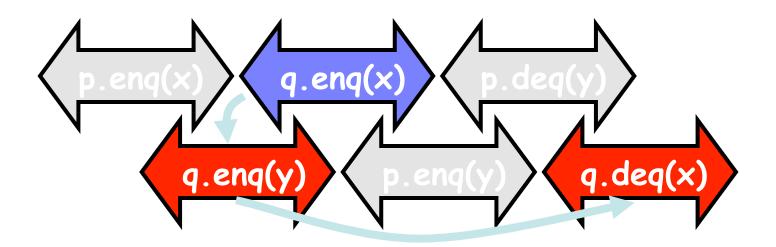
### H|q Sequentially Consistent



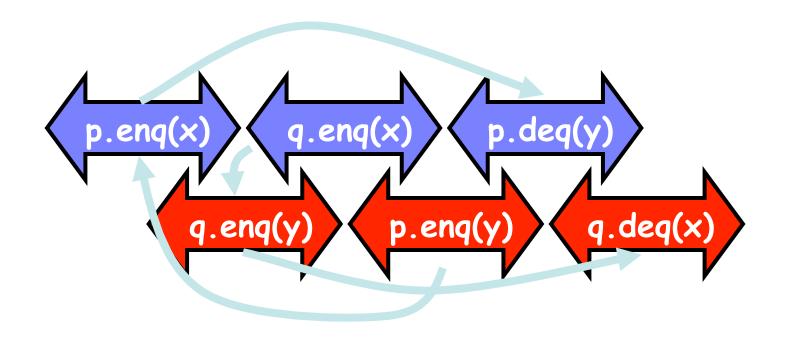
### Ordering imposed by p



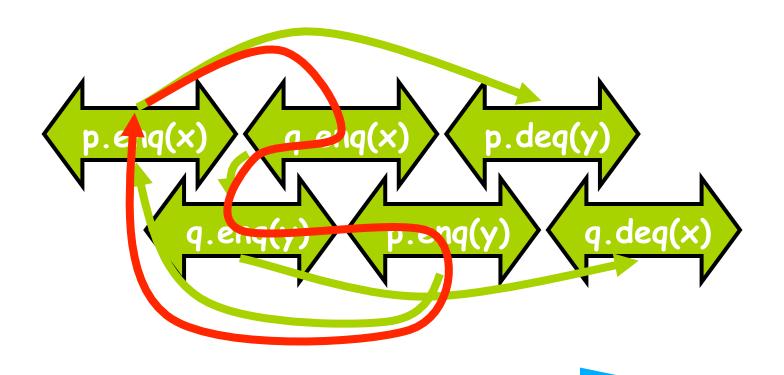
### Ordering imposed by q



### Ordering imposed by both

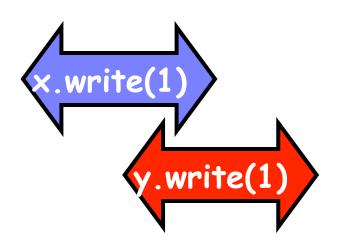


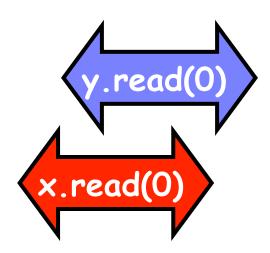
### Combining orders

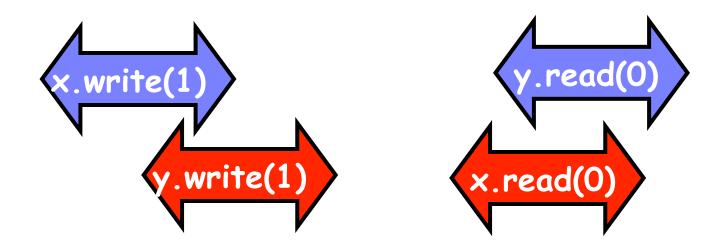


#### Fact

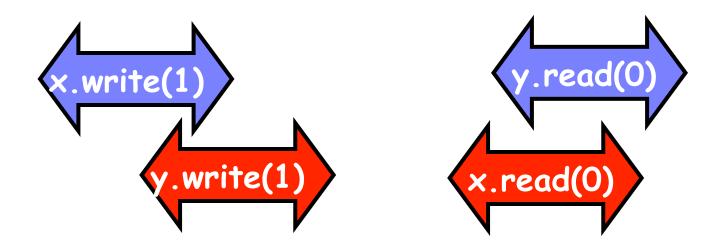
- Most hardware architectures don't support sequential consistency
- Because they think it's too strong
- · Here's another story ...



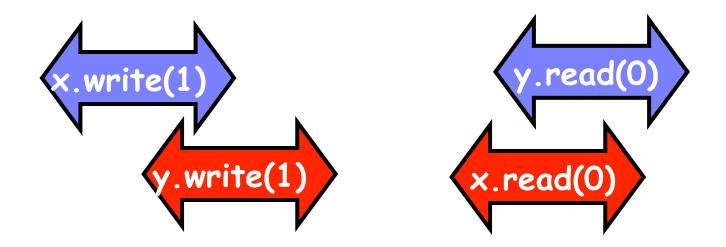




- Each thread's view is sequentially consistent
  - It went first



- Entire history isn't sequentially consistent
  - Can't both go first



- · Is this behavior really so wrong?
  - We can argue either way ...

### Opinion1: It's Wrong

- This pattern
  - Write mine, read yours
- Heart of mutual exclusion
  - Peterson
  - Bakery, etc.
- It's non-negotiable!

## Opinion2: But It Should be Allowed ...

- Many hardware architects think that sequential consistency is too strong
- Too expensive to implement in modern hardware
- · OK if flag principle
  - violated by default
  - Honored by explicit request

### Memory Hierarchy

- On modern multiprocessors, processors do not read and write directly to memory.
- Memory accesses are very slow compared to processor speeds,
- Instead, each processor reads and writes directly to a cache

### Memory Operations

- To read a memory location,
  - load data into cache.
- To write a memory location
  - update cached copy,
  - Lazily write cached data back to memory

### While Writing to Memory

- A processor can execute hundreds, or even thousands of instructions
- Why delay on every memory write?
- Instead, write back in parallel with rest of the program.

### Bottomline ..

- Flag violation history is actually OK
  - processors delay writing to memory
  - Until after reads have been issued.
- Otherwise unacceptable delay between read and write instructions.
- Who knew you wanted to synchronize?

# Who knew you wanted to synchronize?

- Writing to memory = mailing a letter
- Vast majority of reads & writes
  - Not for synchronization
  - No need to idle waiting for post office
- · If you want to synchronize
  - Announce it explicitly
  - Pay for it only when you need it

### Explicit Synchronization

- Memory barrier instruction
  - Flush unwritten caches
  - Bring caches up to date
- · Compilers often do this for you
  - Entering and leaving critical sections
- Expensive

#### Volatile

- In Java, can ask compiler to keep a variable up-to-date with volatile keyword
- Also inhibits reordering, removing from loops, & other "optimizations"

### Real-World Hardware Memory

- Weaker than sequential consistency
- Examples: TSO, RMO, Intel x86...
- But you can get sequential consistency at a price
- · OK for expert, tricky stuff
  - assembly language, device drivers, etc.
- Linearizability more appropriate for high-level software

#### Critical Sections

- · Easy way to implement linearizability
  - Take sequential object
  - Make each method a critical section
- Problems
  - Blocking
  - No concurrency

# Linearizability

- Linearizability
  - Operation takes effect instantaneously between invocation and response
  - Uses sequential specification, locality implies composablity
  - Good for high level objects

## Correctness: Linearizability

- Sequential Consistency
  - Not composable
  - Harder to work with
  - Good way to think about hardware models
- We will use *linearizability* as in the remainder of this course unless stated otherwise

## Progress

- We saw an implementation whose methods were lock-based (deadlockfree)
- We saw an implementation whose methods did not use locks (lock-free)
- How do they relate?

 Minimal progress: in <u>some</u> suffix of H, some pending active invocation has a matching response (some method call eventually completes).

• Minimal progress: in some suffix of H, some pending active invocation has a matching response (some eventually completes).

- Minimal progress: in <u>some</u> suffix of H, some pending active invocation has a matching response (some eventually completes).
- Maximal progress: in every every pending active invocation has a matching response (every method call always completes).

- Minimal progress: in <u>some</u> suffix of H, some pending active invocation has a matching response (some eventually completes).
- Maximal progress: in every every pending active invocation has a matching response (every always completes).

## Progress Conditions

- Deadlock-free: some thread trying to acquire the lock eventually succeeds.
- Starvation-free: every thread trying to acquire the lock eventually succeeds.
- Lock-free: some thread calling a method eventually returns.
- Wait-free: every thread calling a method eventually returns.

## Progress Conditions

Non-Blocking

Blocking

Everyone makes progress

Someone makes progress

Wait-free	Starvation-free
-----------	-----------------

Lock-free

Deadlock-free

### Summary

 We will look at *linearizable blocking* and non-blocking implementations of objects.