Homework #7: MACM-300

Reading: Sipser; Chapter 2; Sections 2.1 and 2.2

Distributed on Feb 22; due on Mar 1 (in class)

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Only submit answers for questions marked with †.

(1) † The following CFG describes regular expressions over the symbols a and b:

$$R \rightarrow R \text{ `}\cup\text{'} R \mid R\ R \mid R \text{ `*'} \mid \text{`('}\ R\ \text{`)'} \mid a \mid b$$

a. Convert this grammar into an unambiguous CFG that resolves ambiguity by assuming that Kleene closure, `*' has the highest priority, followed by concatenation, $RR$, followed by alternation, `∪' . Also assume that each operation associates to the left, which means that RRR should be treated as $(RR)R$ and $R \cup R \cup R$ should be treated as $(R \cup R) \cup R$.

b. Use your unambiguous grammar to parse the string $a \cup b^*b \cup a$ and provide the parse tree.

(2) † Context-free grammars:

a. Show that the following CFG is ambiguous using a string of length 4:

$$S \rightarrow aSbS \mid bSaS \mid \epsilon$$

b. The following CFG generates a regular language. Provide a regular expression that generates the same language as this CFG.

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow c \mid \epsilon \\ B &\rightarrow cbB \mid ca \end{aligned}$$

c. Are the following two context-free grammars equivalent? That is, do the two grammars generate the same language. Give a short precise reason for your answer.

G_1 :

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow c \mid \epsilon \\ B &\rightarrow cbB \mid ca \end{aligned}$$

G_2 :

$$\begin{aligned} S &\rightarrow cAa \\ A &\rightarrow cB \mid B \\ B &\rightarrow bcB \mid \epsilon \end{aligned}$$

d. Provide a context-free grammar for each context-free language below.

1. $L = \{a^i b^i \mid i \geq 1\}$
2. $L = \{a^{2i} b^{3i} \mid i \geq 1\}$
3. $L = \{a^i b^j c^i \mid i, j \geq 1\}$
4. $L = \{a^i b^i c^j \mid i, j \geq 1\}$
5. $L = \{a^i a^i b^j c^j \mid i, j \geq 1\}$

6. $L = \{ua^i wb^i y \mid i, j \geq 1\}$

(3) † Consider the following context-free grammar G.

$$\begin{array}{rcl} S' & \rightarrow & S \\ S & \rightarrow & iSeS \\ S & \rightarrow & iS \\ S & \rightarrow & a \end{array}$$

Now compare it with the following grammar G':

$$\begin{array}{rcl} S' & \rightarrow & S \\ S & \rightarrow & M \mid U \\ M & \rightarrow & iMeM \mid a \\ U & \rightarrow & iS \mid iMeU \end{array}$$

What is the relationship of this grammar to G above?

(4) † Sipser, q2.5

(5) Sipser, q2.7

(6) Sipser, q2.9

(7) Sipser, q2.10

(8) † Sipser, q2.14