# Homework #5: MACM-300

Reading: Notes #2 on course web page

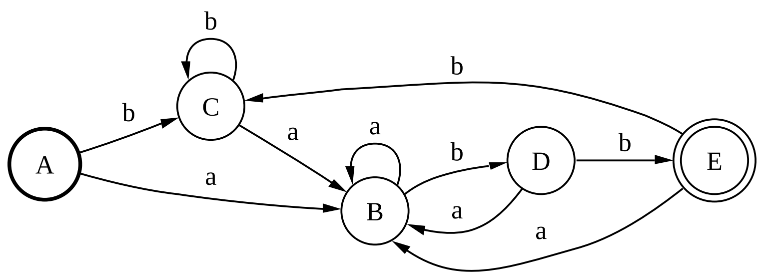
Distributed on Feb 6; due on Feb 13 (in class)

Anoop Sarkar – anoop@cs.sfu.ca

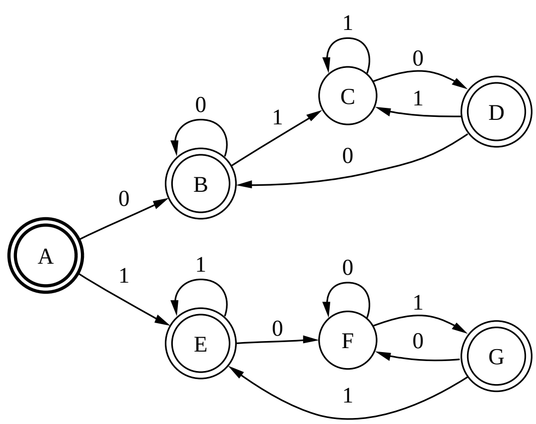
Only submit answers for questions marked with †.

(1) † For each DFA below, provide the minimized DFA using the algorithm in the lecture notes. Show the intermediate steps taken to reach the minimized DFA.

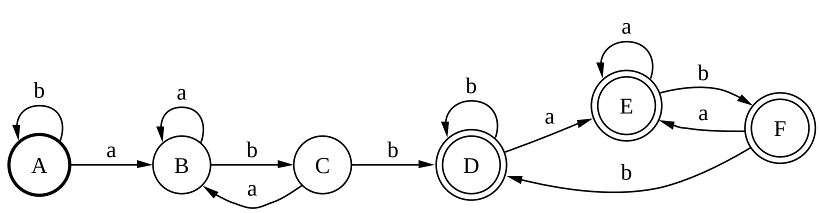
a.




b.




c.




(2) † Two regular expressions can be shown to be equivalent by:

1. Use simple equivalences between regular expressions recursively,
2. Convert each regular expression to an NFA, convert to a DFA, and then minimize the DFA. If the regular expressions are equivalent the minimized DFA will be equivalent upto renaming the states.

Show that the following three regular expressions are equivalent (provide all intermediate steps whichever method you use):

1. $(a \cup b)^*$
2. $(a^* \cup b^*)^*$
3. $((\varepsilon \cup a)b^*)^*$

(3) Sipser, q1.24

(4) Sipser, q1.25

(5) † Sipser, q1.26

(6) † Sipser, q1.27

(7) Sipser, q1.50

(8) † For alphabet $\Sigma$ let us denote the *derivative* of a regular expression $R$ with respect to $a$, where $a \in \Sigma$ as $\frac{dR}{da}$ or equivalently as $D_a[R]$. For any regular expression $R$, $D_a[R]$ is defined recursively using the following rules:

$$D_a[a] = \epsilon \quad (1)$$
$$D_a[b] = \phi, \text{ for } b \in \Sigma,\ b \neq a, \text{ or } b = \epsilon \text{ or } b = \phi \quad (2)$$

If $P$ and $Q$ are regular expressions, then:

$$D_a[P \cup Q] = D_a[P] \cup D_a[Q] \quad (3)$$
$$D_a[PQ] = D_a[P]Q \cup \delta(P)D_a[Q] \quad (4)$$
$$D_a[P^*] = D_a[P]P^* \quad (5)$$

where $\delta(P) = \epsilon$, if $\epsilon \in P$ and $\delta(P) = \phi$, if $\epsilon \notin P$.
The empty set $\phi$, is different from the empty string $\epsilon$, and has the following properties:

$$\phi \cup R = R \cup \phi = R$$
$$\phi R = R\phi = \phi$$

Also, we define derivative $D_s[R]$ of regular expression $R$ with respect to a sequence of symbols $s = a_1, a_2, \ldots, a_r$ as:

$$D_s[R] = D_{a_1,\ldots,a_r}[R] = D_{a_r}[D_{a_1,\ldots,a_{r-1}}[R]]$$

A sequence $s$ of zero length is written as: $D_\epsilon[R] = R$.
The intuition behind the notion of a derivative of a regular expression $R$ with respect to symbol $a$ is that it provides us with a regular expression $R'$ such that the language of $R'$, $L_{R'} = \{y \mid ay \in L_R\}$.
Let $R = (0 \cup 1)^*1$, the derivative $D_a[R]$ for any symbol $a \in \Sigma$, where $\Sigma = \{0, 1\}$ is:

$$\begin{aligned} D_a[R] &= D_a[(0 \cup 1)^*1] \\ &= D_a[(0 \cup 1)^*]\ 1 \cup D_a[1], \text{ since } \epsilon \in (0 \cup 1)^* \quad \text{using (4)} \\ &= D_a[0 \cup 1]\ (0 \cup 1)^*\ 1 \cup D_a[1] \quad \text{using (5)} \\ &= (D_a[0] \cup D_a[1])\ (0 \cup 1)^*\ 1 \cup D_a[1] \quad \text{using (3)} \end{aligned}$$

Putting in $a = 0$, we get:

$$D_0[R] = (\epsilon \cup \phi)(0 \cup 1)^*1 \cup \phi \quad \text{using (1) and (2)}$$

This expressions can be simplified with identities for $\epsilon$ and $\phi$ to get:

$$D_0[R] = (0 \cup 1)^*1 = R$$

We know from our previous definition that $D_\epsilon[R] = R = D_0[R]$. Provide the simplified expressions for the following derivatives of $R = (0 \cup 1)^*1$:

a. $D_1[R]$
b. $D_{10}[R]$
c. $D_{11}[R]$