Homework #4: MACM-300

Reading: Sipser; Chapter 1, Section 1.4

Distributed on Jan 30; due on Feb 6 (in class)

Anoop Sarkar – `anoop@cs.sfu.ca`

Only submit answers for questions marked with †.

(1) Sipser, q1.23

(2) Sipser, q1.29

(3) † Sipser, q1.30

(4) Sipser, q1.46

(5) Sipser, q1.49

Hint: In 1.49.a, the language B can be rewritten using a much simpler description, which is much easier to see as a regular language. It turns out that B has a lot in common with the language defined in Sipser, q1.6.b. *Warning*: B is not *exactly* the same as the language in Sipser q1.6.b (for one thing B has to start with a 1 since in the definition of B, $k \geq 1$ and there is another crucial difference).

(6) † Sipser, q1.55

(7) † Consider the language $L_{\mathcal{R}} = \{w \mid \text{where } w \text{ is a regular expression}\}$. Show that $L_{\mathcal{R}}$ is not a regular language using the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.

Hint: Here's a proof sketch. Follow it to come up with the fully specified (formal) proof. Consider the grouping of expressions using parentheses, e.g. $(((0 \cup 1)^* a) \cup (a \cup b))$. Use closure properties of regular languages (RLs) to simplify the language $L_{\mathcal{R}}$ to a language over the parentheses L_p (due to closure properties of RLs, if $L_{\mathcal{R}}$ is regular, then L_p is regular). Use closure properties of RLs to further simplify L_p to a language L'_p where each open parenthesis "(" precedes each close parenthesis ")". Now use the pumping lemma to show that L'_p is not regular. Since L'_p is not regular, the last step is to use the closure properties of RLs to show that $L_{\mathcal{R}}$ is not regular.