# Homework #2: MACM-300

Reading: Sipser; Chapter 1, Section 1.1

Distributed on Jan 16; due on Jan 23 (in class)

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Only submit answers for questions marked with †.

(1) Sipser, q1.1

(2) Sipser, q1.2

(3) Sipser, q1.3

(4) Sipser, q1.6
*Hint*: Be careful with q1.6.l: start with two DFAs; one DFA for recognizing an even number of `0`s (with any number of `1`s interleaved) and another DFA for recognizing exactly two `1`s (with any number of `0`s interleaved). Then create a single DFA that combines the two DFAs. Note that if the string contains zero `0`s then the string can contain any number of `1`s.

(5) † Provide a DFA for the set of all binary numbers divisible by 4. All strings that are numerically equivalent, e.g. `100`, `0100`, `00100`, etc. should be accepted.

(6) † Provide a DFA that recognizes the the language
$L = \{w \in \{a, b\}* \mid w$ contains an even number of a's and b's $\}$
Note that this means that for every string in the language, if it contains any a's it contains an even number of them and if it contains any b's it contains an even number of them.
Here are some example strings that belong to $L$: $\epsilon, aa, bb, abab, bbaa, aabb, abbababa, \ldots$

(7) † Sipser, q1.12
*Hint*: Come up with an alternative description of the language in terms of a set of strings that can precede or follow another set of strings.

(8) Provide DFAs for $B^{\mathcal{R}}$ and $B$ defined in Sipser, q1.32.

(9) † Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and let $w = w_1 w_2 \ldots w_n$ be a string where each $w_i$ is a member of the alphabet $\Sigma$.
$M$ accepts $w = w_1 \ldots w_n$ if there is a sequence of states $r_0, r_1, \ldots, r_n$ in $Q$ with three conditions:

1. $r_0 = q_0$
2. $\delta(r_i, w_{i+1}) = r_{i+1}$, for $i = 0, \ldots, n-1$, and
3. $r_n \in F$

$\delta^* : Q \times \Sigma^* \rightarrow Q$ is an extended version of the transition function $\delta$ and it is defined on all strings in $\Sigma^*$ as follows:

$$\begin{aligned} \delta^*(q, \varepsilon) &= q \\ \delta^*(q, xa) &= \delta(\delta^*(q, x), a) \text{ where } x \in \Sigma^* \text{ and } a \in \Sigma \end{aligned}$$

Show that if DFA $M$ accepts $w$ by reaching state $f \in F$, then $\delta^*(q_0, w) = f$.