CMPT 825
Natural Language Processing

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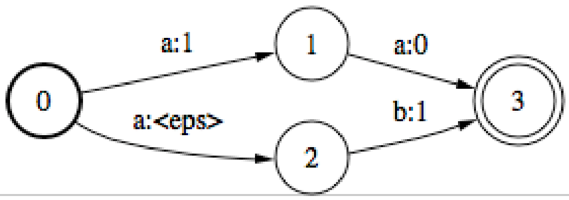
Finite-state transducers

- a : 0 is a notation for a mapping between two alphabets $a \in \Sigma_1$ and $0 \in \Sigma_2$
- Finite-state transducers (FSTs) accept pairs of strings
- Finite-state automata equate to regular languages and FSTs equate to regular relations
- e.g. $L = \{ (x^n, y^n) : n > 0, x \in \Sigma_1 \text{ and } y \in \Sigma_2\}$ is a regular relation accepted by some FST. It maps a string of x's into an equal length string of y's

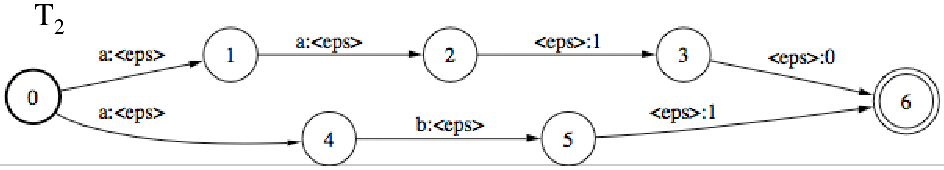
Finite-state transducers

T_1

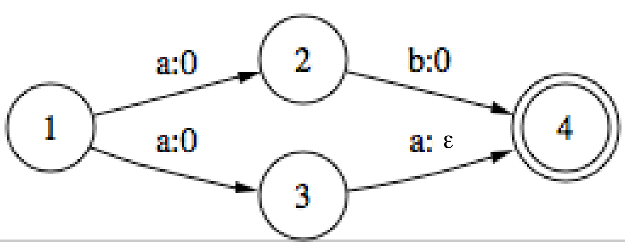


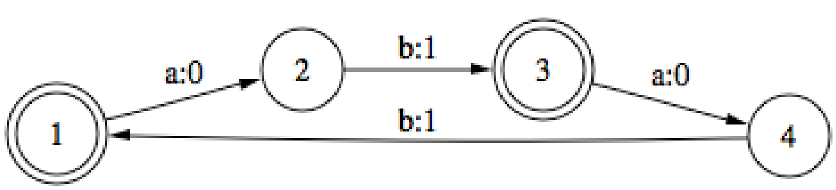
$R(T_1) = R(T_2) = \{ (aa, 10), (ab, 1) \}$

T_2

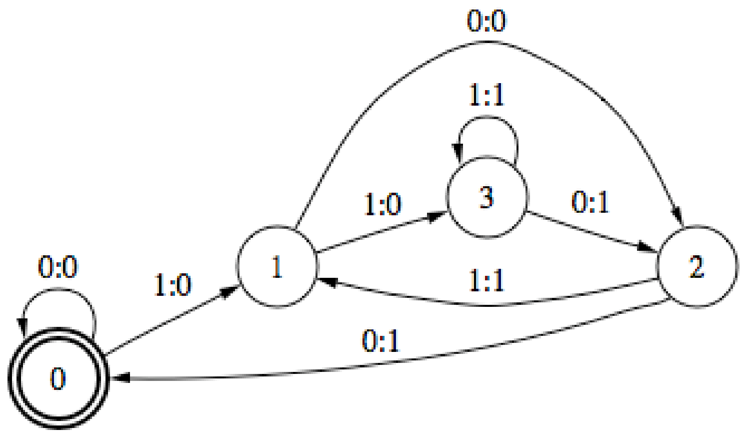


Finite-state transducers





Finite-state transducers



Regular relations

- A generalization of regular languages
- The set of regular relations is:
  - The empty set and (x,y) for all $x, y \in \Sigma_1 \times \Sigma_2$ is a regular relation
  - If R_1, R_2 and R are regular relations then:

    $$R_1 \cdot R_2 = \{(x_1x_2, y_1y_2) \mid (x_1, y_1) \in R_1, (x_2, y_2) \in R_2\}$$

    $$R_1 \cup R_2$$

    $$R^* = \cup_{i=0}^{\infty} R_i$$
  - There are no other regular relations

Finite-state transducers

- Formal definition:
  – Q: finite set of states, $q_0, q_1, ..., q_n$
  – Σ: alphabet composed of input/output pairs $i:o$ where $i \in \Sigma_1$ and $o \in \Sigma_2$ and so $\Sigma \subseteq \Sigma_1 \times \Sigma_2$
  – q_0: start state
  – F: set of final states
  – $\delta(q, i:o)$ is the transition function which returns a set of states

Finite-state transducers: Examples

- (a^n, b^n): map *n a*’s into *n b*’s
- rot13 encryption (the Caesar cipher): assuming 26 letters each letter is mapped to the letter 13 steps ahead (mod 26), e.g. *cipher* → *pvcure*
- reversal of a fixed set of words
- reversal of all strings upto fixed length *k*
- input: binary number *n*, and output: binary number *n+1*
- upcase or lowercase a string of any length
- *Pig latin: *pig latin is goofy* → *igpay atinlay is oofygay*
- *convert numbers into pronunciations,

  e.g. 230.34 two hundred and thirty point three four

Finite-state transducers

- Following relations are cannot be expressed as a FST
  - ($a^n\, b^n$, c^n): because $a^n\, b^n$ is not regular
  - reversal of strings of any length
  - $a^i\, b^j \rightarrow b^j\, a^i$ for any i, j
- Unlike regular languages, regular relations are not closed under intersection
  - ($a^n\, b^*$, c^n) $\cap$ ($a^*\, b^n$, c^n) produces ($a^n\, b^n$, c^n)
  - However, regular relations with input and output of equal lengths **are** closed under intersection

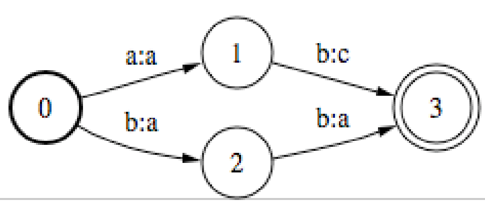
Regular Relations Closure Properties

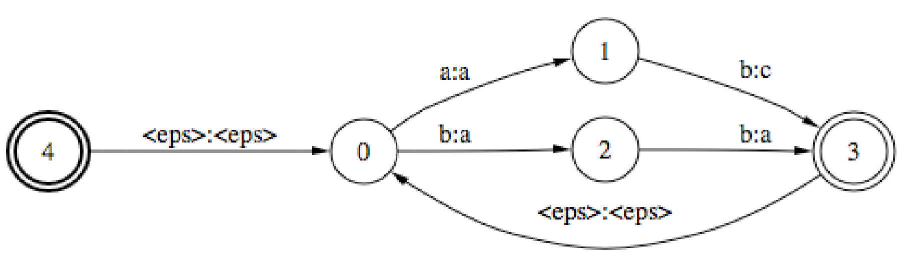
- Regular relations (rr) are ***closed*** under some operations
- For example, if R_1, R_2 are regular relns:
  - union ($R_1 \cup R_2$ results in R_3 which is a rr)
  - concatenation
  - iteration (R_1+ = one or more repeats of R_1)
  - Kleene closure (R_1* = zero or more repeats of R_1)
- However, unlike regular languages, regular relns are not closed under:
  - intersection (possible for equal length regular relns)
  - complement

Regular Relations Closure Properties

- New operations for regular relations:
  - composition
  - project input (or output) language to regular language; for FST t, input language = $\pi_1(t)$, output = $\pi_2(t)$
  - take a regular language and create the identity regular relation; for FSM f, let FST for identity relation be Id(f)
  - take two regular languages and create the cross product relation; for FSMs f & g, FST for cross product is $f \times g$
  - take two regular languages, and mark each time the first language matches any string in the second language

Regular Relation/FST Kleene Closure



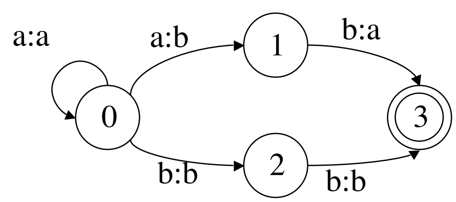


FST Algorithms

- **Compose**: Given two FSTs *f* and *g* defining regular relations R_1 and R_2 create the FST *f o g* that computes the composition: $R_1 \circ R_2$
- **Union**: Given two FSTs *f* and *g* create an FST that computes the union *f+g*
- **Recognition**: Is a given pair of strings accepted by FST *t*?
- **Transduce**: given an input string, provide the output string(s)

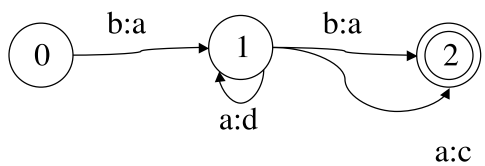
Composing FSTs

T_1:



$a^n\, ab := a^n\, ba$
$a^n\, bb := a^n\, bb$

T_2:

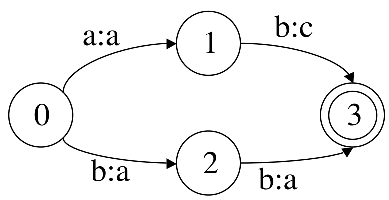


$b\, a^n\, b := a\, d^n\, a$
$b\, a^n\, a := a\, d^n\, c$

What is T_1 composed with T_2, aka $T_1 \circ T_2$?

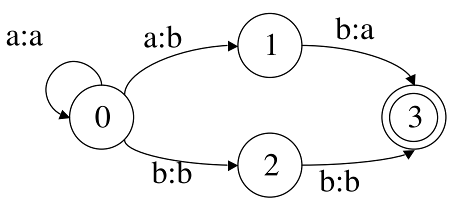
Composing FSTs

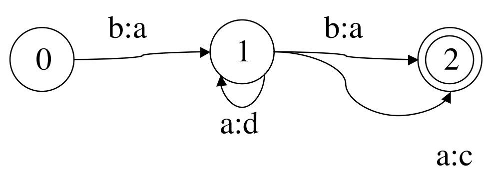
T_1 o T_2:



ab := ac
bb := aa

Composing FSTs





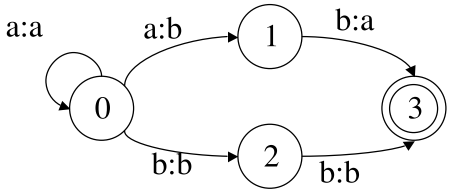
0 1 a : b	0 1 b : a
0 2 b : b	1 2 b : a
2 3 b : b	

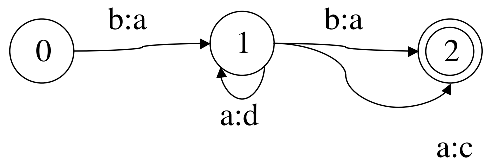
0 0 a : a	1 1 a : d
1 3 b : a	1 2 a : c

(0,0) (1,1) a : a   (0,0) (2,1) b : a
(0,1) (1,2) a : a   (0,1) (2,2) b : a
(2,0) (3,1) b : a   (2,1) (3,2) b : a

(0,1) (0,1) a : d   (1,1) (3,1) b : d
(0,1) (0,2) a : c   (1,1) (3,2) b : c

Composing FSTs





0 1 a : b
0 2 b : b
2 3 b : b

0 1 b : a
1 2 b : a

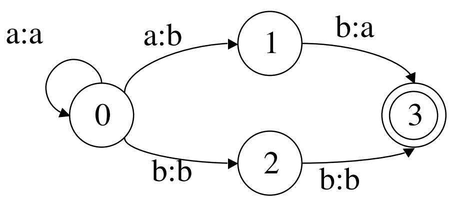
0 0 a : a
1 3 b : a

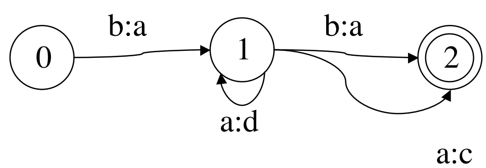
1 1 a : d
1 2 a : c

(0,0) (1,1) a : a   (0,0) (2,1) b : a   (0,1) (0,1) a : d   (1,1) (3,1) b : d
(0,1) (1,2) a : a   (0,1) (2,2) b : a   (0,1) (0,2) a : c   (1,1) (3,2) b : c
(2,0) (3,1) b : a   (2,1) (3,2) b : a

start with pair of final states

Composing FSTs





0 1 a : b
0 2 b : b
2 3 b : b

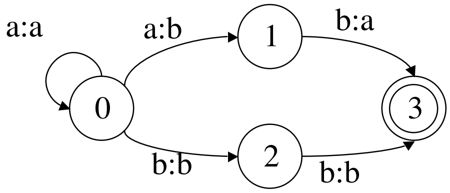
0 1 b : a
1 2 b : a

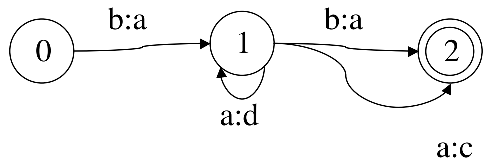
0 0 a : a
1 3 b : a

1 1 a : d
1 2 a : c

(0,0) (1,1) a : a   (0,0) (2,1) b : a   (0,1) (0,1) a : d   (1,1) (3,1) b : d
(0,1) (1,2) a : a   (0,1) (2,2) b : a   (0,1) (0,2) a : c   (1,1) (3,2) b : c
(2,0) (3,1) b : a   (2,1) (3,2) b : a

Composing FSTs





0 1 a : b
0 2 b : b
2 3 b : b

0 1 b : a
1 2 b : a

0 0 a : a
1 3 b : a

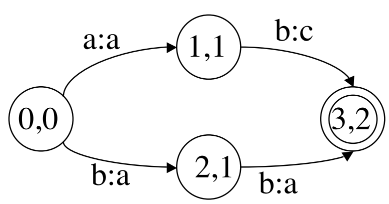
1 1 a : d
1 2 a : c

(0,0) (1,1) a : a   (0,0) (2,1) b : a
(0,1) (1,2) a : a   (0,1) (2,2) b : a
(2,0) (3,1) b : a   (2,1) (3,2) b : a

(0,1) (0,1) a : d   (1,1) (3,1) b : d
(0,1) (0,2) a : c   (1,1) (3,2) b : c

Composing FSTs

T_1 o T_2:



ab := ac
bb := aa

FST Composition

- Input: transducer S and T
- Transducer composition results in a new transducer with states and transitions defined by matching compatible input-output pairs:

  match(s,t) =
  { (s,t) $\rightarrow^{x:z}$ (s',t') : s $\rightarrow^{x:y}$ s' $\in$ S.edges and t $\rightarrow^{y:z}$ t' $\in$ T.edges } $\cup$
  { (s,t) $\rightarrow^{x:\varepsilon}$ (s',t) : s $\rightarrow^{x:\varepsilon}$ s' $\in$ S.edges } $\cup$
  { (s,t) $\rightarrow^{\varepsilon:z}$ (s,t') : t $\rightarrow^{\varepsilon:z}$ t' $\in$ T.edges }
- Correctness: any path in composed transducer mapping *u* to *w* arises from a path mapping *u* to *v* in S and path mapping *v* to *w* in T, for some *v*

Cross-product FST

- For regular languages L_1 and L_2, we have two FSAs, M_1 and M_2

$$M_1 = (\Sigma, Q_1, q_1, F_1, \delta_1)$$
$$M_2 = (\Sigma, Q_2, q_2, F_2, \delta_2)$$

- Then a transducer accepting $L_1 \times L_2$ is defined as:

$$T = (\Sigma, Q_1 \times Q_2, \langle q_1, q_2 \rangle, F_1 \times F_2, \delta)$$
$$\delta(\langle s_1, s_2 \rangle, a, b) = \delta_1(s_1, a) \times \delta_2(s_2, b)$$
$$\text{for any } s_1 \in Q_1, s_2 \in Q_2 \text{ and } a, b \in \Sigma \cup \{\varepsilon\}$$

Subsequential FSTs

- Consider an FST in which for every symbol scanned from the input we can deterministically choose a path and produce an output
- Such an FST is analogous to a deterministic FSM. It is called a **subsequential** FST.
- Subsequential transducers with *p* outputs on the final state is called a ***p*-subsequential** FST
- A subsequential FST with all states as final states is called a **sequential** FST.

Summary

- Finite state transducers specify regular relations
  - Encoding problems as finite-state transducers
- Extension of regular expressions to the case of regular relations/FSTs
- FST closure properties: union, concatenation, composition
- FST special operations:
  - creating regular relations from regular languages (Id, cross-product);
  - creating regular languages from regular relations (projection)
- FST algorithms
  - Recognition, Transduction
  - Determinization, Minimization? (not all FSTs can be determinized)