CMPT-413

Computational Linguistics

Anoop Sarkar

http://www.cs.sfu.ca/~anoop

Why are parsing algorithms important?

- A linguistic theory is implemented in a formal system to generate the set of grammatical strings and rule out ungrammatical strings.

- Such a formal system has computational properties.

- One such property is a simple decision problem: given a string, can it be generated by the formal system *(recognition)*.

- If it is generated, what were the steps taken to recognize the string *(parsing)*.

Why are parsing algorithms important?

- Consider the recognition problem: find algorithms for this problem for a particular formal system.
- The algorithm must be decidable.
- Preferably the algorithm should be polynomial: enables computational implementations of linguistic theories.
- Elegant, polynomial-time algorithms exist for formalisms like CFG

Top-down, depth-first, left to right parsing

$$
\begin{array}{rcl}
S & \rightarrow & NP\ VP \\
NP & \rightarrow & Det\ N \\
NP & \rightarrow & Det\ N\ PP \\
VP & \rightarrow & V \\
VP & \rightarrow & V\ NP \\
VP & \rightarrow & V\ NP\ PP \\
PP & \rightarrow & P\ NP \\
NP & \rightarrow & I \\
Det & \rightarrow & a \mid the \\
V & \rightarrow & saw \\
N & \rightarrow & park \mid dog \mid man \mid telescope \\
P & \rightarrow & in \mid with
\end{array}
$$

Top-down, depth-first, left to right parsing

- Consider the input string: *the dog saw a man in the park*

- S . . . (S (NP VP)) . . . (S (NP Det N) VP) . . . (S (NP (Det the) N) VP) . . . (S (NP (Det the) (N dog)) VP) . . .

- (S (NP (Det the) (N dog)) VP) . . . (S (NP (Det the) (N dog)) (VP V NP PP)) . . . (S (NP (Det the) (N dog)) (VP (V saw) NP PP)) . . .

- (S (NP (Det the) (N dog)) (VP (V saw) (NP Det N) PP)) . . .

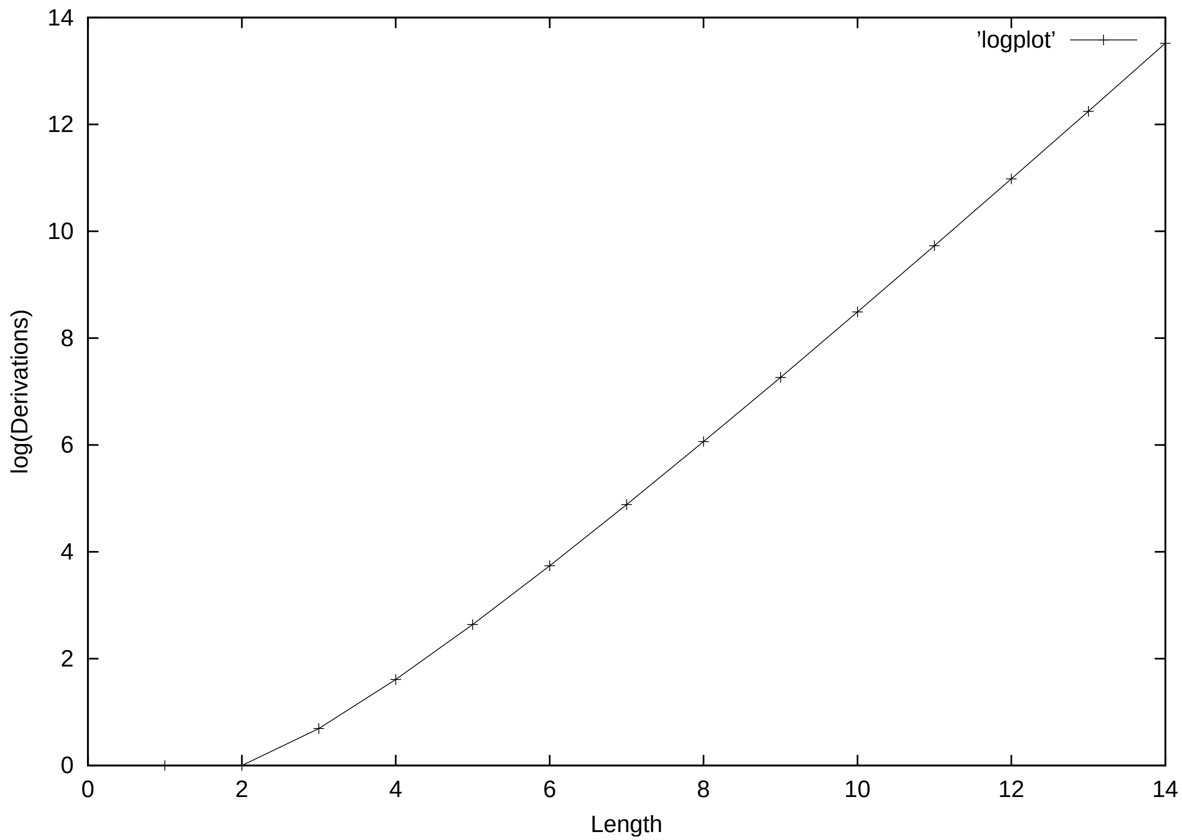
- (S (NP (Det the) (N dog)) (VP (V saw) (NP (Det a) (N man)) (PP (P in) (NP (Det the) (N park)))))

Number of derivations

CFG rules { $S \rightarrow S\ S$, $S \rightarrow a$ }

$n : a^n$	number of parses
1	1
2	1
3	2
4	5
5	14
6	42
7	132
8	429
9	1430
10	4862
11	16796

Number of derivations grows exponentially



$L(G) = a+$ using CFG rules $\{$ `S` $\rightarrow$ `S S` , `S` $\rightarrow$ `a` $\}$

Syntactic Ambiguity: (Church and Patil 1982)

- Algebraic character of parse derivations

- Power Series for grammar for coordination (more general than PPs):

  `NP` $\rightarrow$ `cabbages | kings | NP and NP`

```
NP = cabbages + cabbages and kings
     + 2 (cabbages and cabbages and kings)
     + 5 (cabbages and kings and cabbages and kings)
     + 14 ...
```

CFG Ambiguity

- Coefficients in previous equation equal the number of parses for each string derived from E

- These ambiguity coefficients are Catalan numbers:

$$Cat(n) = \frac{1}{n+1}\binom{2n}{n}$$

- $\binom{a}{b}$ is the *binomial coefficient*

$$\binom{a}{b} = \frac{a!}{(b!(a-b)!)}$$

Catalan numbers

- Why Catalan numbers? Cat(n) is the number of ways to parenthesize an expression of length n with two conditions:

   1. there must be equal numbers of open and close parens

   2. they must be properly nested so that an open precedes a close

Catalan numbers

- For an expression of length n there are a total of $2n$ choose n parenthesis pairs. But $n+1$ of them have the right parenthesis to the left of its matching left parenthesis)(.

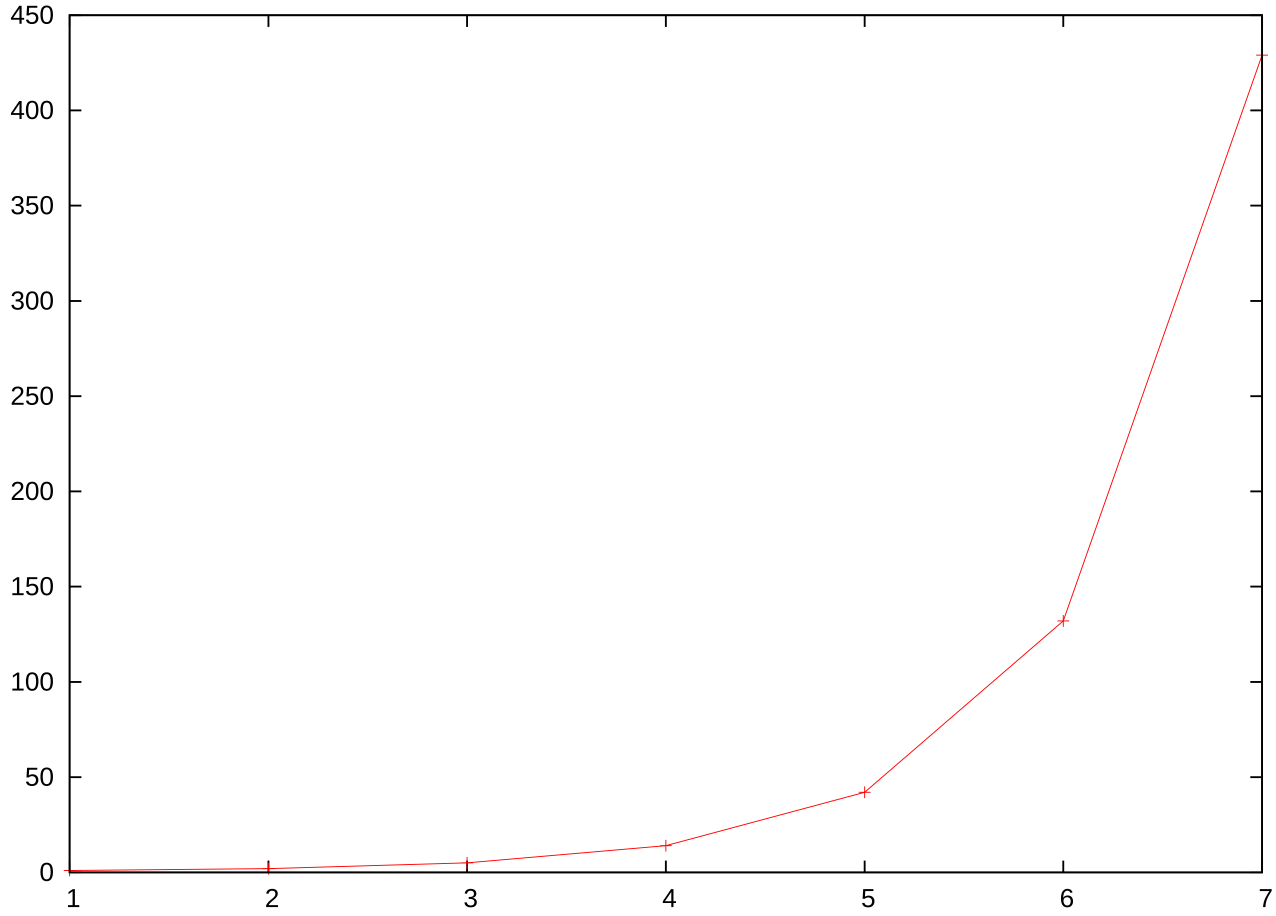
- So we divide $2n$ choose n by $n+1$:

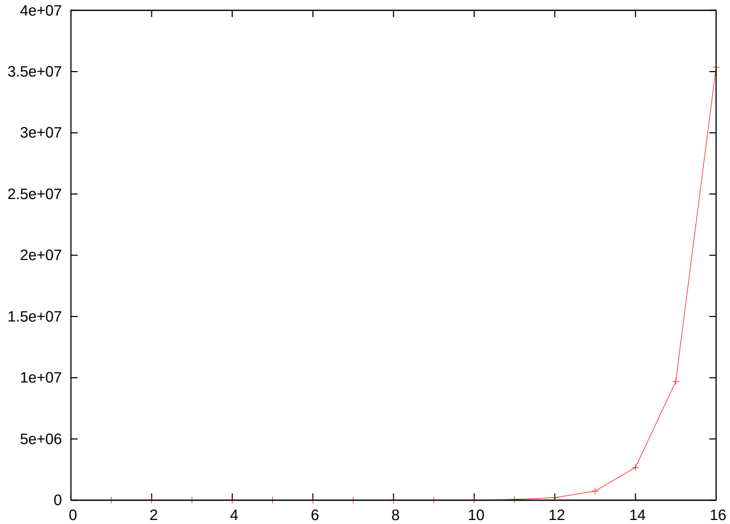
$$Cat(n) = \frac{1}{n+1}\binom{2n}{n}$$

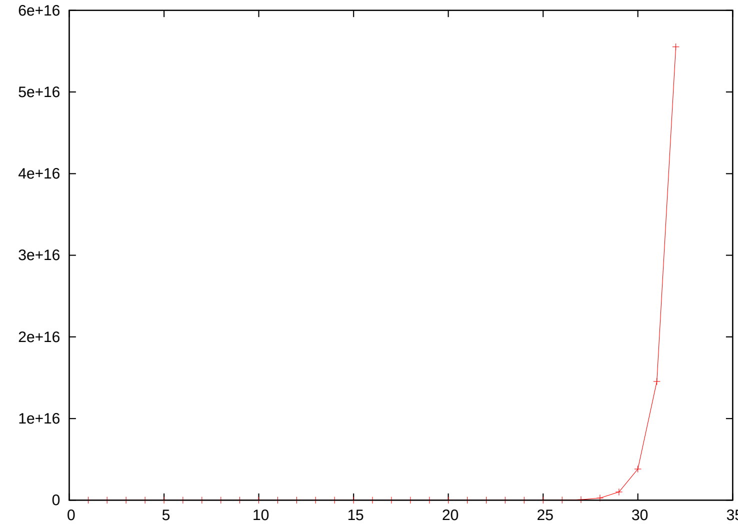
Catalan numbers

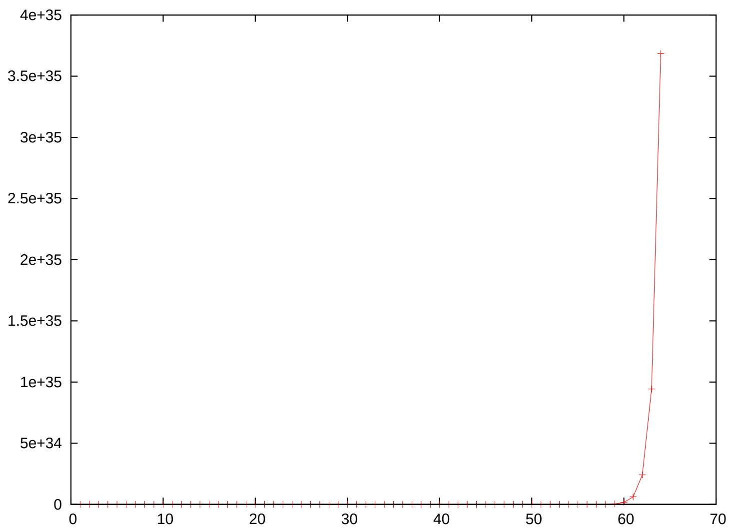
n	catalan(n)
1	1
2	2
3	5
4	14
5	42
6	132
7	429
8	1430
9	4862
10	16796

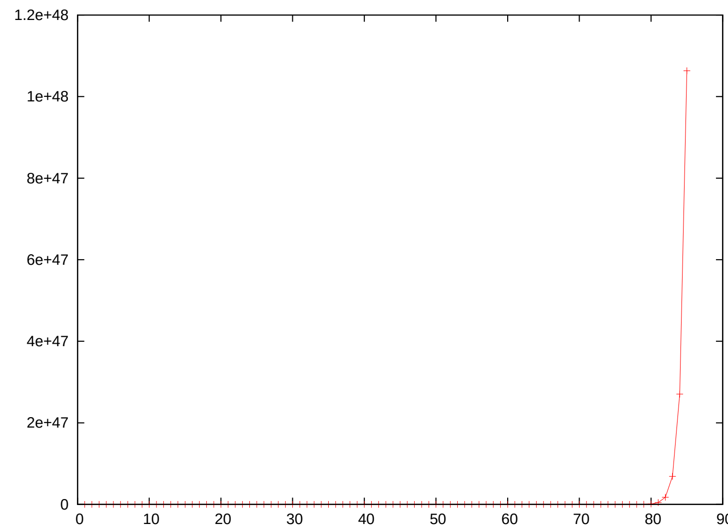
Catalan numbers




4e+07
3.5e+07
3e+07
2.5e+07
2e+07
1.5e+07
1e+07
5e+06
0
0
2
4
6
8
10
12
14
16


6e+16
5e+16
4e+16
3e+16
2e+16
1e+16
0
0
5
10
15
20
25
30
35


4e+35
3.5e+35
3e+35
2.5e+35
2e+35
1.5e+35
1e+35
5e+34
0
0
10
20
30
40
50
60
70


1.2e+48
1e+48
8e+47
6e+47
4e+47
2e+47
0
0
10
20
30
40
50
60
70
80
90

Syntactic Ambiguity

- $Cat(n)$ also provides exactly the number of parses for the sentence: *John saw the man on the hill with the telescope* (generated by the grammar given below, a different grammar will have different number of parses)

$$
\begin{array}{rcl}
S & \rightarrow & NP\ VP \\
NP & \rightarrow & John \mid Det\ N \\
N & \rightarrow & man \mid hill \mid telescope \\
VP & \rightarrow & V\ NP \\
Det & \rightarrow & the
\end{array}
\qquad
\begin{array}{rcl}
VP & \rightarrow & VP\ PP \\
NP & \rightarrow & NP\ PP \\
PP & \rightarrow & P\ NP \\
V & \rightarrow & saw \\
P & \rightarrow & on \mid with
\end{array}
$$

In the above sentence there are 2 PPs, so number of parse trees = Cat($2 + 1$) = 5. With 8 PPs: Cat(9) $= 4862$ parse trees

Syntactic Ambiguity

- Other sub-grammars are simpler. For chains of adjectives:

  cross-eyed pot-bellied ugly hairy professor

  We can write the following grammar, and compute the power series:

$$
\begin{aligned}
ADJP &\rightarrow adj\ ADJP \mid \epsilon \\
ADJP &= 1 + adj + adj^2 + adj^3 + \ldots
\end{aligned}
$$

Syntactic Ambiguity

- Now consider power series of combinations of sub-grammars:

  `S = NP · VP`

  ```
  ( The number of products over sales ... )
  ( is near the number of sales ... )
  ```

- Both the `NP` subgrammar and the `VP` subgrammar power series have Catalan coefficients

Syntactic Ambiguity

- The power series for the `S` $\rightarrow$ `NP VP` grammar is the multiplication:

$$(N \sum_i Cat_i (P N)^i) \cdot (is \sum_j Cat_j (P N)^j)$$

- In a parser for this grammar, this leads to a cross-product:

$$L \times R = \{(l, r) \mid l \in L \,\&\, r \in R \}$$

Syntactic Ambiguity

- A simple change:

```
Is ( The number of products over sales ... )
   ( near the number of sales ... )
```

$$= \texttt{Is}\ N\ \sum_i Cat_i\,(\,P\ N\,)^i)\ \cdot\ (\sum_j Cat_j(\,P\ N\,)^j)$$

$$= \texttt{Is}\ N\ \sum_i \sum_j Cat_i\ Cat_j(\,P\ N\,)^{i+j}$$

$$= \texttt{Is}\ N\ \sum_{i+j} Cat_{i+j+1}\,(\,P\ N\,)^{i+j}$$

Dealing with Ambiguity

- A CFG for natural language can end up providing exponentially many analyses, approx $n!$, for an input sentence of length n

- Much worse than the worst case in the part of speech tagging case, which was n^m for m distinct part of speech tags

- If we actually have to process all the analyses, then our parser might as well be exponential

- Typically, we can directly use the compact description (in the case of CKY, the chart or 2D array, also called a *forest*)

Dealing with Ambiguity

- Solutions to this problem:
  - CKY algorithm: computes all parses in $\mathcal{O}(n^3)$ time. Problem is that worst-case and average-case time is the same.
  - Earley algorithm: computes all parses in $\mathcal{O}(n^3)$ time for arbitrary CFGs,

    $\mathcal{O}(n^2)$ for unambiguous CFGs, and $\mathcal{O}(n)$ for so-called bounded-state CFGs (e.g. $S \rightarrow aSa \mid bSb \mid aa \mid bb$ which generates palindromes over the alphabet a, b).

    Also, average case performance of Earley is better than CKY.
  - Deterministic parsing: only report one parse. Two options: top-down (LL parsing) or bottom-up (LR or shift-reduce) parsing

Shift-Reduce Parsing

- Every CFG has an equivalent pushdown automata: a finite state machine which has additional memory in the form of a stack
- Consider the grammar: $NP \rightarrow Det\ N$, $Det \rightarrow$ *the*, $N \rightarrow$ *dogs*
- Consider the input: *the dogs*
- shift the first word *the* into the stack, check if the top n symbols in the stack matches the right hand side of a rule in which case you can **reduce** that rule, or optionally you can shift another word into the stack

Shift-Reduce Parsing

- reduce using the rule $Det \rightarrow$ *the*, and push Det onto the stack

- shift *dogs*, and then reduce using $N \rightarrow$ *dogs* and push N onto the stack

- the stack now contains *Det, N* which matches the rhs of the rule $NP \rightarrow Det\ N$ which means we can reduce using this rule, pushing NP onto the stack

- If NP is the start symbol and since there is no more input left to shift, we can accept the string

- Can this grammar get stuck (that is, there is no shift or reduce possible at some stage while parsing) on a valid string?

- What happens if we add the rule $NP \rightarrow$ *dogs* to the grammar?

Shift-Reduce Parsing

- Sometimes humans can be “led down the garden-path” when processing a sentence (from left to right)

- Such garden-path sentences lead to a situation where one is forced to backtrack because of a commitment to only one out of many possible derivations

- Consider the sentence:
  The emergency crews hate most is domestic violence.

- Consider the sentence:
  The horse raced past the barn fell

Shift-Reduce Parsing

- Once you process the word *fell* you are forced to reanalyze the previous word *raced* as being a verb inside a *relative clause*: *raced past the barn*, meaning *the horse that was raced past the barn*

- Notice however that other examples with the same structure but different words do not behave the same way.

- For example:
  the flowers delivered to the patient arrived

Earley Parsing

- A *dotted rule* is a way to get around the explicit conversion of a CFG to Chomsky Normal Form
- Since natural language grammars are quite large, and are often modified to be able to parse more data, avoiding the explicit conversion to CNF is an advantage
- A dotted rule denotes that the right hand side of a CF rule has been partially recognized/parsed

Earley Parsing

- $S \rightarrow \bullet NP\ VP$ indicates that once we find an NP and a VP we have recognized an S

- $S \rightarrow NP\ \bullet\ VP$ indicates that we've recognized an NP and we need a VP

- $S \rightarrow NP\ VP\ \bullet$ indicates that we have a complete S

- Consider the dotted rule $S \rightarrow \bullet NP\ VP$ and assume our CFG contains a rule $NP \rightarrow John$
  Because we have such an NP rule we can **predict** a new dotted rule
  $NP \rightarrow \bullet\ John$

Earley Parsing

- If we have the dotted rule: $NP \rightarrow \bullet\ John$ and the next input symbol on our *input tape* is the word *John* we can **scan** the input and create a new dotted rule $NP \rightarrow\ John\ \bullet$
- Consider the dotted rule $S\ \rightarrow\ \bullet NP\ VP$ and $NP \rightarrow\ John\ \bullet$
  Since NP has been completely recognized we can **complete**
  $S\ \rightarrow NP\ \ \bullet\ \ VP$
- These three steps: *predictor*, *scanner* and *completer* form the *Earley parsing algorithm* and can be used to parse using any CFG without conversion to CNF
  Note that we have not accounted for ϵ in the *scanner*

Earley Parsing

- A *state* is a dotted rule plus a span over the input string, e.g. $(S \rightarrow NP \bullet VP, [4, 8])$ implies that we have recognized an NP

- We store all the states in a *chart* – typically, in *chart[i]* we store all states of the form: $(A \rightarrow \alpha \bullet \beta, [i, j])$ *or* states of the form: $(A \rightarrow \alpha \bullet \beta, [j, i])$, where $\alpha, \beta \in (N \cup T)^*$

Earley Parsing

- Note that $(S \rightarrow NP \bullet VP, [0,8])$ implies that in the chart there are two states $(NP \rightarrow \alpha \bullet, [0,8])$ and $(S \rightarrow \bullet NP\ VP, [0,0])$ — this is the *completer* rule, the heart of the Earley parser

- Also if we have state $(S \rightarrow \bullet NP\ VP, [0,0])$ in the chart, then we always *predict* the state $(NP \rightarrow \bullet \alpha, [0,0])$ for all rules $NP \rightarrow \alpha$ in the grammar

Earley Parsing

$$
\begin{array}{rcl}
S & \rightarrow & NP\ VP \\
NP & \rightarrow & Det\ N \mid NP\ PP \mid John \\
Det & \rightarrow & the \\
N & \rightarrow & cookie \mid table \\
VP & \rightarrow & VP\ PP \mid V\ NP \mid V \\
V & \rightarrow & ate \\
PP & \rightarrow & P\ NP \\
P & \rightarrow & on
\end{array}
$$

Consider the input: *0 John 1 ate 2 on 3 the 4 table 5*

What can we predict from the state $(S \rightarrow \bullet\ NP\ VP, [0, 0])$?

What can we complete from the state $(V \rightarrow ate\ \bullet, [1, 2])$?