On clones with distributive subclone lattice

Bulatov A.A. (Ekaterinburg, Russia)

Let A be a finite set and let O_A be the set of all functions on A. An *iterative algebra* is a subset of O_A closed under substitutions of functions, permutations and the identifying of variables and the adding of fictitious variables (see [1]). An iterative algebra containing all the projections (i.e. functions of the form $e_n^i(x_1, \ldots, x_n) = x_i$) is called a *clone*. For any clone C, we denote by $\mathrm{Sub}\, C$ the lattice of subclones of C and by $\mathrm{Sub}'C$ the lattice of iterative subalgebras of C. Here we study clones whose lattice of subclones is distributive. (For brevity we call them distributive clones.) As usual, the clone generated by a set F is denoted by $\langle F \rangle$.

Theorem 1 *For any clone C the lattice* $\mathrm{Sub}\, C$ *is distributive if and only if* $\mathrm{Sub}'C$ *is distributive.*

Every distributive clone is contained in a maximal one. Some well-known clones give examples of maximal distributive clones.

Example 1. Let $A = \{0, 1, 2\}$. The clone generated by functions

$$f_n(x_1, \ldots, x_n) = \begin{cases} 1, & \text{if } x_1 = \ldots = x_{j-1} = x_{j+1} = \ldots = x_n \text{ and} \\ & x_j = 1 \text{ for some } j \in \{1, \ldots, n\} \\ 0 & \text{otherwise} \end{cases}$$

$n \in \mathbb{N}$ ([2]) and the constant function 1 is a maximal distributive clone.

Example 2. Let S be the set of all bijections between proper subsets of A and G a subgroup of the full symmetric group $\mathrm{Sym}\, A$. The clone of all functions preserving $S \cup G$ is distributive if and only if G is transitve and the interval $[G; \mathrm{Sym}\, A]$ of the subgroup lattice of $\mathrm{Sym}\, A$ is distributive.

Two more specific examples are constructed.

References

[1] Mal'tsev A.I. *Post iterative algebras.* Novosibirsk, 1976. (Russian)

[2] Yanov Yu.I, Muchnik A.A. *On existence of k-valued closed classes without a finite basis.* DAN SSSR, 1959, vol.127, no.1, pp.44–46. (Russian)