# Nonmonotonic Reasoning

# and ASP

# Classical Logic and KR

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  *But which?*

- $\neg AtSchool(mary)$

  *But where is she?*

## Classical Logic and KR

- But FOL is limited in the forms of inference that it permits, since the conclusion must be *guaranteed* by the premisses.
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- Commonsense knowledge and reasoning are not like this.
  - Often we want to obtain *plausible* conclusions, ...
  - ...that fill in our incomplete information.

## Generic Statements

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☞ In fact, in commonsense domains, there are almost *no* interesting conditionals that hold universally.

# Types of Defaults

Call a statement of the form "$P$'s are $Q$'s" that allows exceptions a *default*.

Types of defaults:

- Normality: *Birds normally fly*.
- Prototypicality: *The prototypical apple is red*.
- Statistical: *Most students know CPR*.
- Conventional: *Stop for a red light*.
- Persistence: Things tend to remain the same unless something causes a change.
- and many others.

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    $$\forall x\ P(x) \wedge \neg E_{x_1}(x) \wedge \cdots \wedge \neg E_{x_n}(x) \supset Q(x)$$

    doesn't work since
    - we can't list all exceptional conditions $E_{x_1}, \ldots, E_{x_n}$, and
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☞ Need theories of how *plausible* conclusions may be drawn from uncertain, partial evidence.

# Nonmonotonic Reasoning

In the notation of FOL:

Monotonic: If $\Gamma \vdash \alpha$ then $\Gamma, \Delta \vdash \alpha$.
Non-monotonic: If $\Gamma \vdash \alpha$, possibly $\Gamma, \Delta \not\vdash \alpha$.

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  - E.g. conclude that a bird flys, *unless you have a reason to believe otherwise*
- A rule *P's are (normally, usually) Q's* is called a *default*.
- The goal is to account for *default reasoning* (not to be confused with *Default Logic*, which is a specific approach).

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☞ We'll use ASP, which is strongly related to Default Logic, to formalise default reasoning.

# Representing Defaults in ASP

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  A: Want to conclude that a bird flies if
  - there is no reason to believe that it doesn't fly,
  - i.e. it is *consistent* that it flies.
- Can express this with the rule:

$$fly(X) \leftarrow bird(X), not\ \neg fly(X)$$

## Example

- Consider:
  $bird(tweety).$
  $bird(opus).$
  $\neg fly(opus).$
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  $fly(X) \leftarrow bird(X), not\ \neg fly(X).$
- Obtain one answer set containing
  $fly(tweety).$

## Another Example

- Consider:
  $q(rn).$
  $r(rn).$
  $p(X) \leftarrow q(X), not\ \neg p(X).$
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  $\{q(rn),\ r(rn),\ p(rn)\}$
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- What to believe?

  First approximation:

  Credulous: Choose an extension arbitrarily
  Skeptical: Intersect the extensions.

# Interacting Defaults

- Consider where we have that birds fly but penguins don't fly: $penguin(opus)$.

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- Aside: What if we replaced the rule by:
  $fly(X) \leftarrow bird(X), \neg penguin(X), not\ \neg fly(X).$ ?

# A Similar Problem

- "Penguins are birds" is represented by a strict rule.
- A transitivity involving only defaults is handled the same way. E.g.:
  - typically topics in ASP are topics in KR:
    $kr(X) \leftarrow asp(X), not\ \neg kr(X)$
  - typically topics in KR are interesting:
    $int(X) \leftarrow kr(X), not\ \neg int(X)$
  - typically topics in ASP are not interesting:
    $\neg int(X) \leftarrow asp(X), not\ int(X)$
- For $asp(co)$, get two answer sets with $int(co)$ and $\neg int(co)$.
  - (Aside: co = "combinatorial optimization")
- Fix by replacing the second rule by

$$int(X) \leftarrow kr(X), not\ asp(X), not\ \neg int(X)$$

# Other Interacting Defaults

- Default may interact in other ways. E.g.:
  - typically birds fly:
    $fly(X) \leftarrow bird(X), not\ \neg fly(X)$
  - typically baby birds don't fly:
    $\neg fly(X) \leftarrow bbird(X), not\ fly(X)$
    (Of course, if baby birds *never* fly, then one would use
    $\neg fly(X) \leftarrow bbird(X)$.)
- For $bbird(huey)$ we get two answer sets, and fix by using:

$$fly(X) \leftarrow bird(X), not\ bbird(X), not\ \neg fly(X)$$

# A Methodology for Interacting Defaults

Problem: One difficulty with the "fix" for avoiding unwanted answer sets is that all exceptional conditions are added to a rule.

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Methodology: Express a default like "birds normally fly" as:

$$fly(X) \leftarrow bird(X), not\ ab_{bf}(X), not\ \neg fly(X)$$

- I.e. "birds that are not known to be abnormal wrt flight, fly"
- Since penguins are abnormal in this regard, we would also have

  $$ab_{bf}(X) \leftarrow penguin(X).$$

- In fact we could also assert:

  $$ab_{bf}(X) \leftarrow \neg fly(X)$$

  and then have the simple default:

  $$fly(X) \leftarrow bird(X), not\ ab_{bf}(X)$$

# Another Example: Flying Birds

Assuming that (very!) exceptional penguins may fly:

$$bird(X) \leftarrow penguin(X).$$

$$fly(X) \leftarrow bird(X), not\ ab_{bf}(X), not\ \neg fly(X).$$
$$\neg fly(X) \leftarrow penguin(X), not\ ab_{pf}(X), not\ fly(X).$$
$$fly(X) \leftarrow$$
$$penguin(X), very\_fit(X), not\ ab_{fitp}(X), not\ \neg fly(X).$$

$$ab_{bf}(X) \leftarrow penguin(X).$$
$$ab_{pf}(X) \leftarrow very\_fit(X).$$