Nonmonotonic Reasoning

and ASP
Classical Logic and KR

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- $\exists x \text{ Loves}(\text{mary}, x)$.
  
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  *But where is she?*
Classical Logic and KR

- But FOL is limited in the forms of inference that it permits, since the conclusion must be *guaranteed* by the premisses.
  - E.g. ask: Is Ralph, a raven, black?
  - To *derive* this information, we can (effectively) only reason from facts about Ralph, or general knowledge about ravens.
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• Commonsense knowledge and reasoning are not like this.
  • Often we want to obtain *plausible* conclusions, . . .
  • . . . that fill in our incomplete information.
Generic Statements

*Observe:* Most of the properties of objects or topics in everyday life hold *normally* or *usually* or *in general*.

For example:

- “Ravens are black”.
- “Medication \( x \) is used to treat ailment \( y \)”
- “John goes for coffee at 10:00”.

In fact, in commonsense domains, there are almost no interesting conditionals that hold universally.
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  *Every raven? Albinos? A raven you’re told isn’t black?*
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- and similarly for everyday topics including trees, pens, games, weddings, coffee, temporal persistence, etc.
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Types of Defaults

Call a statement of the form “$P$’s are $Q$’s” that allows exceptions a default.

Types of defaults:

• **Normality**: *Birds normally fly.*
• **Prototypicality**: *The prototypical apple is red.*
• **Statistical**: *Most students know CPR.*
• **Conventional**: *Stop for a red light.*
• **Persistence**: Things tend to remain the same unless something causes a change.
• and many others.
Nonmonotonic Reasoning

General Goal:
Given that $P$’s are normally $Q$’s holds and that $P(a)$ is true, want to conclude $Q(a)$ unless there is a good reason not to.

Classical inference clearly isn’t sufficient.

For example, listing exceptional conditions:

$$\forall x \ P(x) \land \neg E_x^1(x) \land \cdots \land \neg E_x^n(x) \supset Q(x)$$

doesn’t work since

• we can’t list all exceptional conditions $E_x^1, \ldots, E_x^n$, and
• we don’t want to have to prove $\neg E_x^1(a), \ldots, \neg E_x^n(a)$ in order to conclude $Q(a)$.

Need theories of how plausible conclusions may be drawn from uncertain, partial evidence.
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In the notation of FOL:

Monotonic: If $\Gamma \vdash \alpha$ then $\Gamma, \Delta \vdash \alpha$.
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- In nonmonotonic theories, an inference may depend on a lack of information.
  - E.g. conclude that a bird flys, *unless you have a reason to believe otherwise*
- A rule *P’s are (normally, usually) Q’s* is called a *default*.
- The goal is to account for *default reasoning* (not to be confused with *Default Logic*, which is a specific approach).
Nonmonotonic Reasoning

There are several major approaches to NMR.

- **Closed World Assumption**: A fact is assumed to be false if it cannot be shown to be true.

- **Default Logic**: Add rules of the form $\alpha: \beta \gamma$ to classical logic. Roughly: If $\alpha$ is true and $\beta$ is consistent then conclude $\gamma$.

- **Autoepistemic Logic**: Roughly, if something were true, I’d know it.

- **Circumscription**: Formalise the notion that a predicate applies to as few individuals as possible. Then can write $\forall x (P(x) \land \neg ab(x) \supset Q(x))$.

- **Nonmonotonic Inference Relations**: Formalise a notion of nonmonotonic inference $\alpha \not\rightarrow \beta$.

We’ll use ASP, which is strongly related to Default Logic, to formalise default reasoning.
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Representing Defaults in ASP

- We’ll assume that we have classical negation in our rules. (Recall that we can encode classical negation in ASP.)
  - Consider the assertion “birds (normally) fly”.
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  - Q: What does this mean?

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- Q: What does this mean?
  - A: Want to conclude that a bird flies if
    - there is no reason to believe that it doesn’t fly,
    - i.e. it is consistent that it flies.
- Can express this with the rule:
  
  \[
  \text{fly}(X) \leftarrow \text{bird}(X), \text{not } \neg \text{fly}(X)
  \]
Example

• Consider:
  \[\text{bird(tweety)}.\]
  \[\text{bird(opus)}.\]
  \[\neg\text{fly(opus)}.\]
  \[\text{fly}(X) \leftarrow \text{bird}(X), \text{not } \neg\text{fly}(X).\]
Example

• Consider:
  \[ bird(tweety) \].
  \[ bird(opus) \].
  \[ \neg fly(opus) \].
  \[ fly(X) \leftarrow bird(X), \text{not } \neg fly(X) \].

• Obtain one answer set containing
  \[ fly(tweety) \].
Another Example

- Consider:
  
  \[ q(rn). \]
  
  \[ r(rn). \]
  
  \[ p(X) \leftarrow q(X), \text{not} \neg p(X). \]
  
  \[ \neg p(X) \leftarrow r(X), \text{not} p(X). \]
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- Obtain two answer sets
  
  \[ \{ q(rn), r(rn), p(rn) \} \]
  
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• What to believe?

  First approximation:
  
  **Credulous:** Choose an extension arbitrarily
  **Skeptical:** Intersect the extensions.
Interacting Defaults

- Consider where we have that birds fly but penguins don’t fly: 
  \[\text{penguin}(\text{opus}).\]
  \[\text{fly}(X) \leftarrow \text{bird}(X), \text{not } \neg \text{fly}(X).\]
  \[\neg \text{fly}(X) \leftarrow \text{penguin}(X), \text{not } \text{fly}(X).\]
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- Obtain two answer sets, one containing \( \text{fly}(\text{opus}) \) and one containing \( \neg \text{fly}(\text{opus}) \).

- **Problem**: Have an unwanted “transitivity”
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  **Solution:** Block by replacing the first rule by

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• Aside: What if we replaced the rule by:
  \[
  \begin{align*}
  \text{fly}(X) \leftarrow \text{bird}(X), \neg penguin(X), \text{not } \neg \text{fly}(X).
  \end{align*}
  \]
A Similar Problem

• “Penguins are birds” is represented by a strict rule.

• A transitivity involving only defaults is handled the same way.
  E.g.:
  • typically topics in ASP are topics in KR:
    \[ kr(X) \leftarrow asp(X), \text{not } \neg kr(X) \]
  • typically topics in KR are interesting:
    \[ int(X) \leftarrow kr(X), \text{not } \neg int(X) \]
  • typically topics in ASP are not interesting:
    \[ \neg int(X) \leftarrow asp(X), \text{not } int(X) \]

• For \( asp(co) \), get two answer sets with \( int(co) \) and \( \neg int(co) \).
  • (Aside: \( co = \text{“combinatorial optimization”} \))

• Fix by replacing the second rule by
  \[ int(X) \leftarrow kr(X), \text{not } asp(X), \text{not } \neg int(X) \]
Other Interacting Defaults

- Default may interact in other ways. E.g.:
  - typically birds fly:
    \[ \text{fly}(X) \leftarrow \text{bird}(X), \text{not } \neg \text{fly}(X) \]
  - typically baby birds don't fly:
    \[ \neg \text{fly}(X) \leftarrow \text{bbird}(X), \text{not } \text{fly}(X) \]
    (Of course, if baby birds never fly, then one would use
    \[ \neg \text{fly}(X) \leftarrow \text{bbird}(X). \])
  - For \text{bbird}(huey) we get two answer sets, and fix by using:
    \[ \text{fly}(X) \leftarrow \text{bird}(X), \text{not } \text{bbird}(X), \text{not } \neg \text{fly}(X) \]
A Methodology for Interacting Defaults

Problem: One difficulty with the “fix” for avoiding unwanted answer sets is that all exceptional conditions are added to a rule.

- May become cumbersome, difficult to read, or error prone.
A Methodology for Interacting Defaults

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• May become cumbersome, difficult to read, or error prone.

Methodology: Express a default like “birds normally fly” as:

\[ \text{fly}(X) \leftarrow \text{bird}(X), \text{not } ab_{bf}(X), \text{not } \neg\text{fly}(X) \]

• I.e. “birds that are not known to be abnormal wrt flight, fly”

• Since penguins are abnormal in this regard, we would also have

\[ ab_{bf}(X) \leftarrow \text{penguin}(X). \]

• In fact we could also assert:

\[ ab_{bf}(X) \leftarrow \neg \text{fly}(X) \]

and then have the simple default:

\[ \text{fly}(X) \leftarrow \text{bird}(X), \text{not } ab_{bf}(X) \]
Another Example: Flying Birds

Assuming that (very!) exceptional penguins may fly:

\[
\begin{align*}
bird(X) & \leftarrow penguin(X). \\
fly(X) & \leftarrow bird(X), \text{ not } ab_{bf}(X), \text{ not } \neg fly(X). \\
\neg fly(X) & \leftarrow penguin(X), \text{ not } ab_{pf}(X), \text{ not } fly(X). \\
fly(X) & \leftarrow \\
& \quad penguin(X), \text{ very } \_\text{fit}(X), \text{ not } ab_{fitp}(X), \text{ not } \neg fly(X). \\
ab_{bf}(X) & \leftarrow penguin(X). \\
ab_{pf}(X) & \leftarrow \text{ very } \_\text{fit}(X).
\end{align*}
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