Formal Argumentation

CMPT 411/721

Introduction

Argumentation:

. . . the study of processes *"concerned with how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held"*.
[Bench-Capon and Dunne, *Argumentation in AI*, AIJ 171, 2007]

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Formal models of argumentation are concerned with

- representing an argument
- representing the relationship between arguments
- solving conflicts between the arguments ("acceptability")

Why Argumentation?

Agent Reasoning

- Internal reasoning:
  - Reasoning about beliefs, goals, intentions, etc. often is defeasible
- Interaction with other agents:
  - Information exchange, negotiation, collaboration, ...

Why Argumentation?

Application areas

- Medical diagnosis and treatment
- Legal reasoning
  - Interpretation
  - Evidence / crime investigation
- Single and multi-agent defeasible reasoning about conflicting goals, intentions, etc.
- Decision making
- Policy design
- ...

Why Argumentation?

Systems

- PARMENIDES system: Facilitates structured arguments over a proposed course of action
- IMPACT project: Argumentation toolbox for supporting deliberations about public policy
- ASPIC+: Fully developed system; applications to business, medicine
- Decision support systems, etc.

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Steps in the Argumentation Process

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  $KB = \{s,\ r,\ w,\ s \rightarrow \neg r,\ r \rightarrow \neg w,\ w \rightarrow t\}$

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- Resolve conflicts: A_1, A_3
- Draw conclusions: $\neg r$, t

Example

Argument a:

1. John Smith is a public person.
2. $\therefore$ It's ok to publish an article about his public life

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Argument a:

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Argument b:

1. John Smith has retired from politics.
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Argument b:

1. John Smith has retired from politics.
2. ∴ He is no longer a public person.

Form:

$$b \longrightarrow a$$

Terminology: Argument *b* *attacks* *a*.

Example

Argument c:

1. John Smith continues to write articles and blog.
2. ∴ He is a public person.

Example

Argument c:

1. John Smith continues to write articles and blog.
2. ∴ He is a public person.

Form:

$$c \longrightarrow b \longrightarrow a$$

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Argument a:

1. Richard is a Quaker and Quakers are pacifists.
2. $\therefore$ Richard is a pacifist.

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Argument b:

1. Richard is a (US) Republican and Republicans are not pacifists.
2. $\therefore$ Richard is not a pacifist.

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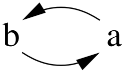

Abstract Argumentation

- Begin with a knowledge base
- Generate arguments from the knowledge base
- Identify conflicts between arguments
- Abstract from internal structure

☞ Resolve conflicts

☞ Draw conclusions

Abstract Argumentation

- Originally due to Dung [Dung, 1995].
  - Still the most active research area in argumentation.
- Main idea: Abstract away from the logical content of arguments and only consider the *relation* between arguments.
  - Select subsets of arguments respecting certain criteria as the accepted arguments.
  - Key question: What are these criteria?
- Obtain:
  - Simple, yet powerful, formalism
- Downside: Lots of competing semantics

Argumentation Framework

Definition

An *argumentation framework* (AF) is a pair (A, R) where

- A is the set of arguments
- $R \subseteq A \times A$ represents the *attacks* relation

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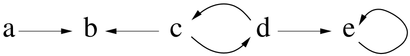

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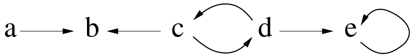

Key Issue:

What are the accepted arguments?

☞ I.e. What is the appropriate subset $S \subseteq A$ to accept?

General Approach: Basic Definitions

Given an AF $F = (A, R)$:

- A set $S \subseteq A$ is *conflict free* if for every $a, b \in S$, we have $(a, b) \notin R$.

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  - S is conflict free in F
  - each $a \in S$ is defended by S in F
    - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists $c \in S$, such that $(c, b) \in R$.

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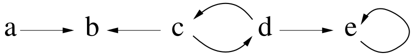
Intuition: For a set of arguments to be accepted, it must be

- coherent (i.e. conflict free), and
- if any argument in the set is challenged by a counterargument, an argument in the set offers a counterargument to that counterargument.

Example

In:



- Conflict-free:
- Admissable:

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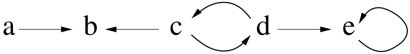

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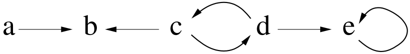

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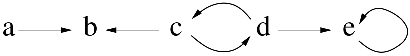

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☞ However, it seems funny to have admissable sets that don't include a, since a isn't attacked.

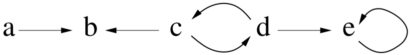
A Simple Approach: Grounded Extensions

Given an AF $F = (A, R)$. The unique *grounded extension* of F is defined as the outcome S of the following procedure:

1. Set $S \leftarrow \emptyset$.
2. Select an argument a which is not attacked; if no such argument exists, return S.
3. $S \leftarrow S \cup \{a\}$.
4. Remove from F all arguments attacked by a, together with their "attacks" relations.
5. Go to Step 2.

Grounded Extensions

Example:



Grounded Extensions

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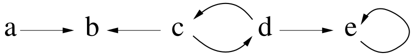

There is one grounded extension $\{a\}$

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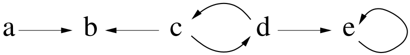

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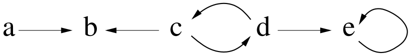

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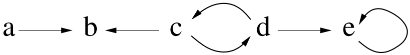

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- Grounded extensions are clearly conflict-free and admissable.
- Grounded extensions are unique.
- However, the results are quite weak.

Preferred Extensions

Definition

A set $S \subseteq A$ is a *preferred extension* iff

- S is admissable in (A, R)
- for each admissable $T \subseteq A$, $S \not\subseteq T$.

☞ That is, a preferred extension is a $\subseteq$-maximal admissable set.

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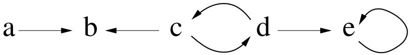

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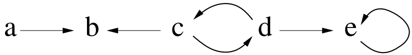

Preferred extensions: $\{a, c\}, \{a, d\}$

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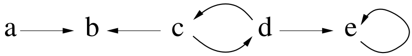

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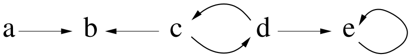


Stable extension: $\{a, d\}$.

Some Properties

For any AF $F = (A, R)$ the following hold:

- Each stable extension of F is admissible
- Each stable extension of F is also a preferred extension

Also

- Grounded and preferred extensions always exist.
- A stable extension may not exist.
- If (A, R) has no cycles then there is a single grounded, stable (and thus preferred) extension.

Decision Problems on AFs

Credulous Acceptance

Given AF $F = (A, R)$ and $a \in A$:

Is a contained in *at least one* extension of F?

Skeptical Acceptance

Given AF $F = (A, R)$ and $a \in A$:

Is a contained in *every* extension of F?

Complexity Results

Credulous reasoning

Theorem

1. *CRED is in P for the grounded semantics*
2. *CRED is NP-complete for admissability*
3. *CRED is NP-complete for the preferred semantics*
4. *CRED is NP-complete for the stable semantics*

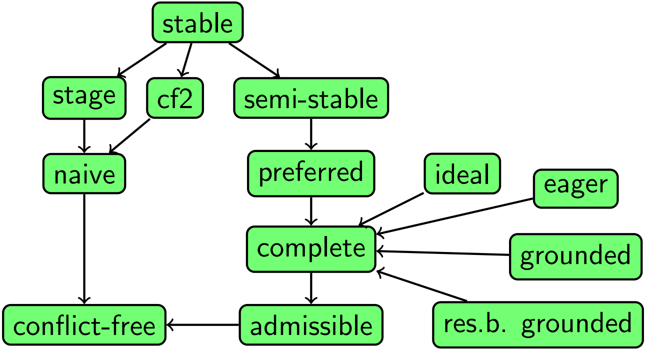
Complexity Results

Skeptical reasoning

Theorem

1. *SKEPT is in P for the grounded semantics*
2. *SKEPT is computationally trivial for admissability.*
3. *SKEPT is coNP-hard for the preferred semantics*
4. *SKEPT is coNP-complete for the stable semantics.*

Other Semantics



- An arrow from σ to τ specifies that each σ-extension is also a τ-extension.
- (Diagram from Stefan Woltran)

Argumentation Based on Classical Logic

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Idea:

- An argument involves premisses and a conclusion.
- In general, the connection between premisses and conclusion could involve analogical, causal, inductive, normative, or any other type of inference.
- In the B&H approach, classical logic is used.
- We'll consider a restriction of classical logic to what B&H call *simple logic*.

Simple Logic

Simple logic is a restriction of propositional logic.

- The language is formed from a set of atomic sentences $\{a, b, \ldots\}$.
- The language consists of literals (atoms or their negation) and rules of the form

$$l_1 \wedge \cdots \wedge l_n \rightarrow l$$

  where $l_1, \ldots, l_n, l$ are all literals.
- The only inference rule is modus ponens.
    - E.g. from $p,\ p \rightarrow s$, conclude s
      from $\neg s,\ p \rightarrow s$, don't conclude anything

Introduction

- An *argument* is a pair $\langle \Psi, \alpha \rangle$, where Ψ is a minimal consistent sets of formulas that entails α.
- Ψ is the *support* and α is the *claim* of the argument.
- E.g. $\langle \{a,\ b,\ a \wedge b \rightarrow c\},\ c \rangle$
  $\langle \{a,\ a \rightarrow b,\ b \rightarrow c\},\ c \rangle$

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- Each undercut is itself an argument and so in turn may be undercut, and so on.
- This leads to an *argument graph*, a synthesis of arguments and counterarguments.
- Basic approach: Systematically explore the space of arguments to show that a given claim does or does not hold.

Approach

Definition:
Let Δ be a set of formulas in a logic (here, *simple logic*).
An *argument* is a pair $\langle \Psi, \alpha \rangle$ such that

1. $\Psi \not\vdash \bot$
2. $\Psi \vdash \alpha$
3. Ψ is a minimal subset of Δ satisfying 2.

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If $A = \langle \Psi, \alpha \rangle$ is an argument, then

- A is an *argument* for α
- Ψ is a *support* for α
- α is the *claim* of the argument

Argumentation Definition: Condition 1

Condition 1: $\Psi \nvdash \bot$

- For Condition 1 of the definition, want to exclude arguments of the form

  $\langle \{a, \neg a\}, b \rangle$

  - E.g. exclude

    John is a student
    John is not a student,
    ―――――――――――――
    Therefore today is Tuesday

Argumentation Definition: Condition 2

Condition 2: $\Psi \vdash \alpha$

- That is, Ψ gives a reason for accepting α.

Argumentation Definition: Condition 3

Condition 3: Ψ is a minimal subset of Δ satisfying 2.

- In Condition 3, we exclude irrelevant arguments.
- So, for example, exclude
  $\langle\{a, a \rightarrow b, c\}, b\rangle$
  - E.g. exclude:
    John is a grad student
    Grad student are students
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    ____
    Therefore John is a student
- Hence each $\beta \in \Psi$ is an *essential* part of the argument for α.
  - So the claim α can be attacked by attacking any $\beta \in \Psi$.

Example

Let $\Delta = \{a,\ a \rightarrow b,\ b \rightarrow d,\ c \rightarrow \neg b,\ c,\ d,\ d \rightarrow b,\ \neg a,\ \neg c\ \}$

The following are possible arguments:

$\langle\{a, a \rightarrow b\},\ b\rangle$
$\langle\{a, a \rightarrow b, b \rightarrow d\},\ d\rangle$
$\langle\{c, c \rightarrow \neg b\},\ \neg b\rangle$
$\langle\{d, d \rightarrow b\},\ b\rangle$
$\langle\{\neg a\},\ \neg a\rangle$
$\langle\{\neg c\},\ \neg c\rangle$

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$\langle\{\neg c\},\ \neg c\rangle$

The following are not arguments:

$\langle\{a, \neg a\},\ a\rangle$
$\langle\{a, c, c \rightarrow b\},\ b\rangle$

Counterarguments

An argument that disagrees with another is a *counterargument*.
The two most important notions of conflict between arguments are:

- An *undercut* for an argument $\langle \Psi, \alpha \rangle$ is an argument $\langle \Psi', \neg\phi \rangle$ where $\Psi \vdash \phi$.
- An argument $\langle \Psi, \beta \rangle$ is a *rebuttal* for an argument $\langle \Psi', \alpha \rangle$ iff α is equivalent to $\neg\beta$.

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Example

Let $\Delta = \{a, a \rightarrow b, b \rightarrow d, c, c \rightarrow \neg a, c \rightarrow \neg b\}$.
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Let $\Delta = \{a, a \rightarrow b, b \rightarrow d, c, c \rightarrow \neg a, c \rightarrow \neg b\}$.
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- $\langle \{c, c \rightarrow \neg a\}, \neg a \rangle$ is an undercut for $\langle \{a, a \rightarrow b\}, b \rangle$.
- $\langle \{c, c \rightarrow \neg b\}, \neg b \rangle$ is an undercut for $\langle \{a, a \rightarrow b, b \rightarrow d\}, d \rangle$.
- $\langle \{a\}, a \rangle$ is a simple rebuttal for $\langle \{c, c \rightarrow \neg a\}, \neg a \rangle$.

A More Realistic Example

p	Simon Jones is a Member of Parliament
$p \to \neg q$	If Simon Jones is a Member of Parliament then we need not keep quiet about details of his private life
r	Simon Jones just resigned from the House of Commons
$r \to \neg p$	If Simon Jones just resigned from the House of Commons then he is not a Member of Parliament
$\neg p \to q$	If Simon Jones is not a Member of Parliament then we need to keep quiet about details of his private life

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Consider:

$$\langle \{p, p \rightarrow \neg q\}, \neg q \rangle$$

An undercut is:

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A rebuttal is:

$$\langle \{r, r \rightarrow \neg p, \neg p \rightarrow q\}, q \rangle.$$

Constructing Argument Graphs

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Descriptive Argument Graphs

Given a set of arguments and counterarguments, form the abstract (directed) graph.

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☞ In either case, can then use techniques from the first part to come up with conclusions (i.e. supported claims).

Descriptive Argument Graphs

Example 1

A_1 : "The flight is low cost and luxury, therefore it is a good flight"

A_2 : "If a flight is low cost then it can't be luxury. Since it is low cost, it's not luxury"

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A_2 : "If a flight is low cost then it can't be luxury. Since it is low cost, it's not luxury"

Thus A_2 attacks A_1:

A2 ⟶ A1

Descriptive Argument Graphs

Example 2

Assertions:

- *bp*(*high*) – patient has high blood pressure
- *ok*(*bb*) – it's ok to give a betablocker
- *ok*(*di*) – it's ok to give a diuretic
- *give*(*bb*) – prescribe a betablocker
- *give*(*di*) – prescribe a diuretic
- *symp*(*emph*) – patient shows signs of emphysema

Descriptive Argument Graphs

Informally:

A_1: The patient has high blood pressure, it's ok to give them diuretics and it's not ok to give them betablockers.
$\therefore$ Give a diuretic

A_2: The patient has high blood pressure, it's ok to give them a betablocker and it's not ok to give them a diuretic.
$\therefore$ Give a betablocker

A_3: The patient has symptoms of emphysema. Since if someone has symptoms of emphysema it's not ok to give a betablocker,
$\therefore$ It's not ok to give them betablockers.

Descriptive Argument Graphs

Formally:

A_1: $\langle \{ bp(high), ok(di), \neg give(bb)$
$\quad bp(high) \wedge ok(di) \wedge \neg give(bb) \rightarrow give(di) \},$
$give(di) \rangle$

A_2: $\langle \{ bp(high), ok(bb), \neg give(di)$
$\quad bp(high) \wedge ok(bb) \wedge \neg give(di) \rightarrow give(bb) \},$
$give(bb) \rangle$

A_3: $\langle \{ symp(emph), symp(emph) \rightarrow \neg ok(bb) \}, \neg ok(bb) \rangle$

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Formally:

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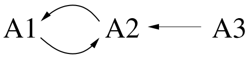
$bp(high) \wedge ok(di) \wedge \neg give(bb) \rightarrow give(di)\},$
$give(di)\rangle$

A_2: $\langle\{bp(high), ok(bb), \neg give(di)$
$bp(high) \wedge ok(bb) \wedge \neg give(di) \rightarrow give(bb)\},$
$give(bb)\rangle$

A_3: $\langle\{symp(emph), symp(emph) \rightarrow \neg ok(bb)\}, \neg ok(bb)\rangle$

Form:



Generative Argument Graphs

Let Δ be a simple logic knowledge base.

Define:

$$\mathit{Arguments}(\Delta) = \{\langle\Psi, \alpha\rangle \mid \Psi \subseteq \Delta \text{ and } \langle\Psi, \alpha\rangle \text{ is a simple argument } \}$$

$$\mathit{Attacks}(\Delta) = \{(A; B) \mid A, B \in \mathit{Arguments}(\Delta) \text{ and } A \text{ is an undercut of } B\}$$

Aside: In simple logic, any rebuttal is also an undercut, so we just need to consider undercuts.

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$\{(A; B) \mid A, B \in Arguments(\Delta)$ and A is an undercut of $B\}$

Aside: In simple logic, any rebuttal is also an undercut, so we just need to consider undercuts.

An *exhaustive graph* for Δ is an argument graph $G = (\mathcal{A}, \mathcal{R})$ where

- $\mathcal{A}$ is $Arguments(\Delta)$ and
- $\mathcal{R}$ is $Attacks(\Delta)$.

Example

$$\Delta_1 \;=\; \{a,\; b,\; c,\; \neg a,\; \neg b,\; \neg c,\; a \rightarrow \neg b,\; b \rightarrow \neg c,\; c \rightarrow d\}$$

$A_1 : \langle \{a,\; a \rightarrow \neg b\},\; \neg b \rangle$

$A_2 : \langle \{b,\; b \rightarrow \neg c\},\; \neg c \rangle$

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Form:

A1 ⟶ A2 ⟶ A3

Example

$$\Delta_1 \; = \; \{a,\ b,\ c,\ \neg a,\ \neg b,\ \neg c,\ a \rightarrow \neg b,\ b \rightarrow \neg c,\ c \rightarrow d\}$$

$A_1 : \langle \{a,\ a \rightarrow \neg b\},\ \neg b \rangle$

$A_2 : \langle \{b,\ b \rightarrow \neg c\},\ \neg c \rangle$

$A_3 : \langle \{c,\ c \rightarrow d\},\ d \rangle$

Form:

$$\mathrm{A1} \longrightarrow \mathrm{A2} \longrightarrow \mathrm{A3}$$

- ☞ There is one (grounded or preferred or stable) extension with A_1 and A_3, supporting the claims $\neg b$ and d.

Example

$$\Delta_2 \;=\; \{a,\ b,\ c,\ \neg a,\ \neg b,\ \neg c,\ a \to \neg b,\ b \to \neg c,\ c \to \neg a\}$$

$A_1 : \langle\{a,\ a \to \neg b\},\ \neg b\rangle$

$A_2 : \langle\{b,\ b \to \neg c\},\ \neg c\rangle$

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Example

$$\Delta_2 \ = \ \{a,\ b,\ c,\ \neg a,\ \neg b,\ \neg c,\ a \to \neg b,\ b \to \neg c,\ c \to \neg a\}$$

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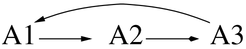

Example

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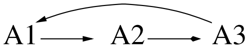

- ☞ There is one grounded or preferred extension, the empty set, and no stable extension.
- ☞ In either case no claims are supported.

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