Chapter 3

Semantics

Topics

- Introduction
- Static Semantics
- Attribute Grammars
- Dynamic Semantics
- Operational Semantics
- Axiomatic Semantics
- Denotational Semantics

Introduction

- Language implementors
  - Understand how all the constructs of the language are form and their intended effect when executed.
- Language users
  - Determine how to encode a possible solution of a problem (program) using the reference manual of the programming language.
- Less knowledge of how to correctly define the semantics of a language.

Introduction

- Well-designed programming language
  - Semantics should follow directly from syntax.
  - Form of a statement should strongly suggest what the statement is meant to accomplish.
- Definition of a programming language
  - Complete: semantics and syntax are fully defined.
- A language should provides a variety of different constructs, each one with a precise definition.

Introduction

- Language manuals
  - Definition of semantics is given in ordinary natural language.
  - Construct
    - Syntax: a rule (or set of rules) from a BNF or other formal grammar.
    - Semantics: a few paragraphs and some examples.

Introduction

- Natural language description
  - Ambiguous in its meaning
    - Different readers come away with different interpretations of the semantics of a language construct.
- A method is needed for giving a readable, precise, and concise definition of the semantics of an entire language.

Static Semantics

- BNFs cannot describe all of the syntax of programming languages.
  - Some context-specific parts are left out.
- Is there a form to generate $L=\{a^n b^n c^n\}$ using a context-free grammar or a BNF?
- An attempt:

<string> ::= <aseq> <bseq> <c seq>
<a seq> ::= a | <aseq> a
<b seq> ::= b | <bseq> b
<c seq> ::= c | <c seq> c

$L'=\{a^k b^m c^n \mid k=1, m=1, n=1\}$
No context-free grammar generates L

Static Semantics

- Some problems have nothing to do with "meaning" in the sense of run-time behavior
  - They are concern about the legal form of the program.
  - Static semantics refers to type checking and resolving declarations.
  - Examples:
    - All variables must be declared before they are referenced
    - Ada: the name on the end of a procedure must match the procedure's name
    - Both sides of an assignment must be of the same type.

Static Semantics

- Earliest attempts to add semantics to a programming language
- Add extensions to the BNF grammar that defined the language.
  - Given a parse tree for a program, additional information could be extracted from that tree.

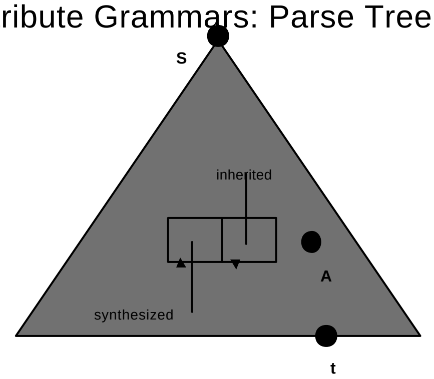
Attribute Grammars: Basic Concepts

- A context-free grammar extended to provide context-sensitivity information by appending attributes to each node of a parse tree.
- Each distinct symbol in the grammar has associated with it a finite, possibly empty, set of attributes.
  - Each attribute has a domain of possible values.
  - An attribute may be assigned values from its domain during parsing.
  - Attributes can be evaluated in assignments and conditions.

Attribute Grammars: Generalities

- Two classes of attributes:
  - Synthesized attribute
    - Gets its value from the attributes attached to its children (subtree below the node).
    - Used to pass semantic information up a parse tree.
  - Inherited attribute
    - Gets its value from the attributes attached to the parent (subtree above the node).
    - Used to pass semantic information down and across a tree.

Attribute Grammars: Parse Tree



Attribute Grammar Definition

- Associate some functions to compute the value of the attributes with each production in the grammar.
- These local definitions associated with each production of the grammar define the values of the attributes for all parse trees.
- Given the definitions and a parse tree, algorithms exist to compute the attributes of all the nodes in the three.

Attribute Grammars

- Starting with the underlying context-free grammar *G=<N,T,P,S>*
- For every production *p* in *P*
  - Number of terminal and nonterminal symbols in string *a* : *n(p)*.
    - If *a* is the empty string, then *n(p)=0*.
    - Sometimes each symbol of a production will be considered individually.
      - For all production *pÎ P* : *A? a* or p_0? p_1,p_2,… $p_{n(p)}$

Attribute Grammars

- Augment the context-free grammar by attributes and semantic rules.
- Set of attributes: *At*.
  - For each attribute *a ÎAt*: associate a set of values *Domain(a).*
  - An attribute is just a name for a set of values
- Set of attributes: two disjoint classes:
  - Inherited attributes ***In*** and the synthesized attributes ***Syn*** (*At=InÈSyn* and *InÇSyn=Æ*).

Attribute Grammars: attributes

- *There is a set of attributes At(x)ÌAt to every grammar symbol xÎNÈT*
  - *At(x) can be seen as additional information about the symbol x.*
- *Set*
  - *In(x) = { a ÎAt(x) | a Î In }*
  - *Syn(x) = { aÎAt(x) | aÎ Syn }*
  - *Requirements:*
    - *In(S)= Æ (start symbol can inherit no information)*
    - *For all tÎT, Syn(t)= Æ (there is no structure beneath a terminal from which to synthesize information)*

Attribute Grammars: rules

- Same attribute can be associated with different symbols appearing in the same grammar rule.
  - Example: *S? AB*, all could inherit attribute `int` associated to them: *In(S)=In(A)=In(B)={**int**}*.
  - It is impossible to consider the set of attributes associated with all the symbols of a production without losing track of which attributes appear more than once.
  - More confusing: productions that have a nonterminal appearing more than once, as in *S? ASA*.

Attribute Grammars: attribute occurrences

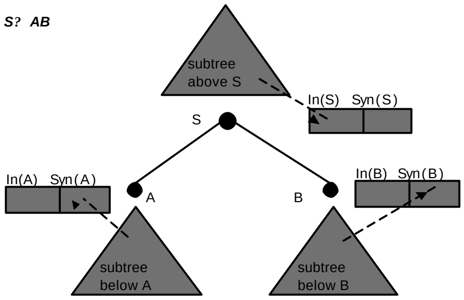
- *Attribute occurrence* of a rule *p* is an ordered pair of attributes and natural number *<a,j>* representing the attribute *a* at position *j* in production *p.*
  - Particular rule *p ÎP* an attribute occurrence at *j* will be written p_j.*a*.
  - Set of attribute occurrences for a production *p* is defined: *AO(p) = {* p_j*.a | a ÎAt(*p_j*), 0£ j £ n(p) }*

Attribute Grammars: attribute occurrences

- Set of attribute occurrences for a rule is divided into two disjoint subsets.
  - *Defined occurrences* for a production *p*:
    - *DO(p) = { p_0.s | s Î Syn(p_0)} È { p_j.i | i Î In(p_j), 1£ j £ n(p) }*
      - *In a parse tree, the set UO(p) represents the information flowing into the node of the parse tree labeled p_0*
  - *Used occurrences for a production p:*
    - *UO(p) = { p_0.i | i Î In(p_0)} È { p_j.s | s Î Syn(p_j), 1£ j £ n(p) }*
      - *In a parse tree, the set DO(p) represents the information flowing out flowing into the node of the parse tree labeled p_0*

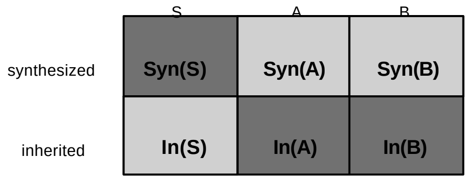
Attribute Grammars: flow of attribute occurrences

Rule: ***S? AB***



Attribute Grammars: used attribute occurrences

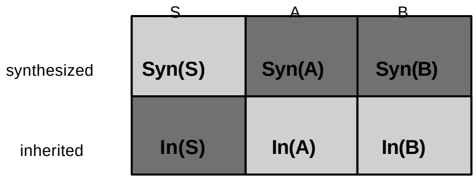
- Used attribute occurrences (the information flowing in) are *In(S), Syn(A),* and *Syn(B).*



	S	A	B
synthesized	**Syn(S)**	**Syn(A)**	**Syn(B)**
inherited	**In(S)**	**In(A)**	**In(B)**

Attribute Grammars: defined attribute occurrences

- Defined attribute occurrences (the information flowing out) are *Syn(S), In(A),* and *In(B).*



	S	A	B
synthesized	**Syn(S)**	**Syn(A)**	**Syn(B)**
inherited	**In(S)**	**In(A)**	**In(B)**

Attribute Grammars: semantic function

- Semantic function f$_{p,v}$.
  - For every attribute occurrence *v Î DO(p)*
  - Defined values for attributes in *DO(p)* in terms of the values of the attributes in *UO(p).*
  - Produces a value for the attribute *a* from values of the attributes of *UO(p).*
  - There is no requirement that all the attribute occurrences of *UO(p)* are used by f$_{p,v}$.
  - *Dependency set* (*D*$_{p,v}$.) of f$_{p,v:}$ is the set of attribute occurrences used (subset of *UO(p)*)
  - *D*$_{p,v}$ could be empty
    - Value of the attribute: computed without any other additional information. The function f$_{p,v}$ is a constant.

Attribute Grammar

- An attribute grammar as a context-free grammar with two disjoint sets of attributes (inherited and synthesized) and semantic functions for all defined attribute occurrences.

Attribute Grammar: binary digits example

- Context-free grammar that generates strings of binary digits.

  p: B ® D
  q: B ® D B
  r: D ® 0
  s: D ® 1

	B	D
synthesized	pos, val	val
inherited		pow

- Attributes:
  - *val*: accumulate the value of the binary numbers
  - *pow* and *pos*: keep track of the position and the power of 2.

Attribute Grammar: binary digits example

- Compute the defined and the used occurrences for each production
- The defined occurrences is the set of synthesized attributes of the LHS plus the set of inherited attributes of all the grammar symbols of the RHS.

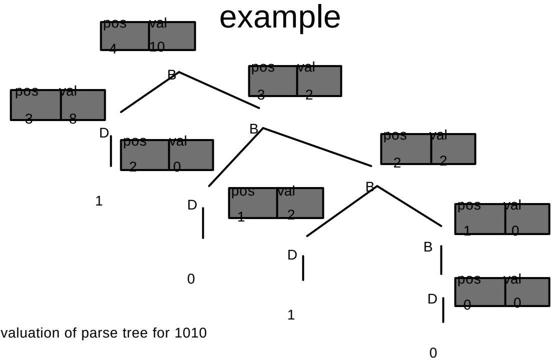
	Defined	Used
p	B.pos, B.val, D.pow	D.val
q	B_1.pos, B_1.val, D.pow	B_2.pos, B_2.val, D.val
r	D.val	D.pow
s	D.val	D.pow

Attribute Grammar: binary digits example

- Function definitions for the eight defined attribute occurrences.

```
p:   B ® D
          B.pos := 1
          B.val := D.val
          D.pow := 0
q:   B1 ® D B2
          B1.pos := B2.pos+1
          B1.val := B2.val+D.val
          D.pow := B2.pos
r:   D ® 0
          D.val := 0
s:   D ® 1
          D.val := 2^D.pow
```

Attribute Grammar: binary digits example



Evaluation of parse tree for 1010

Dynamic Semantics

- Semantics of a programming language is the definition of the *meaning* of any program that is syntactically valid.
- intuitive idea of programming meaning: "whatever happens in a (real or model) computer when the program is executed."
  - A precise characterization of this idea is called *operational semantics.*

Dynamic Semantics

- Another way to view programming meaning is to start with a formal specification of what a program is supposed to do, and then rigorously prove that the program does that by using a systematic series of logical steps.
  - This approach evokes the idea of *axiomatic semantics*.

Dynamic Semantics

- A third way to view the semantics of a programming language is to define the meaning of each type of statement that occurs in the (abstract) syntax as a state-transforming mathematical function.
  - The meaning of a program can be expressed as a collection of functions operating on the program state.
  - This approach is called *denotational semantics.*

Dynamic Semantics: advantages and disadvantages

- Operational Semantics
  - Advantage of representing program meaning directly in the code of a real (or simulated) machine.
  - Potential weakness, since the definition of semantics is confined to a particular architecture (either real or abstract).
    - Virtual machine also needs a semantic description, which adds complexity and can lead to circular definitions.

Dynamic Semantics: advantages and disadvantages

- *Axiomatic semantics is useful in the exploration of formal properties of programs.*
  - *Programmers who must write provably correct programs from a precise set of specification are particularly well-served by this semantic style.*
- *Denotational semantics is valuable because its functional style brings the semantic definition of a language to a high level of mathematical precision*
  - *Language designers obtain a functional definition of the meaning of each language construct that is independent of any particular machine architecture.*

Operational Semantics

- Provides a definition of program meaning by simulating the program’s behavior on a machine model that has a very simple (through not necessarily realistic) instruction set and memory organization.
- Definition of the virtual computer can be described using an existing programming language or a virtual computer (idealized computer).

Operational Semantics: process

- Change in the state of the machine (memory, registers, etc) defines the meaning of the statement.
- The operational semantics of a high-level language can be described using a virtual computer.
  - A pure hardware interpreter is too expensive.
  - A pure software interpreter has also problems:
    - Machine-dependent
    - Difficult to understand
  - A better alternative: a complete computer simulation.

Operational Semantics: process

- The process:
  - Identify a virtual machine (an idealized computer).
  - Build a translator (translates source code to the machine code of an idealized computer).
  - Build a simulator for the idealized computer.
- Operational semantics is sometimes called *transformational semantics*, if an existing programming language is used in place of the virtual machine.

Operational Semantics: automaton

- Automaton could be used as a virtual machine:
  - More complex that the simple automata models used in the study of syntax and parsing
- Automaton has
  - Internal state that corresponds to the internal state of the program when it its executing;
    - The state contains all the values of the variables, the executable program, and various system-defined housekeeping data structures.

Operational Semantics: automaton

- A set of formally defined operations used to specify how the internal state of the automaton may change,
  - Corresponds to the execution of one instruction in the program.
- A second part of the definition specifies how a program text is translated into an initial state for the automaton
  - From this initial state, the rules defining the automaton specify how the automaton moves from state to state until a final state is reached.

Operational Semantics: process

- Example:

Pascal statement

```
for i := x to y do
     begin
          ...
     end
```

Operational Semantics (lower level)

```
mov i, r1
mov y, r2
jmpifless(r2,r1,out)
```

Operational Semantics

```
            i := x
loop: if i > y  goto out
            ...
            i := i + 1
            goto loop
out: ...
```

Operational Semantics: evaluation

- Advantages:
  - May be simple, intuitive for small examples/
  - Good if used informally.
  - Useful for implementation.
- Disadvantages:
  - Very complex for large programs.
  - Depends on programming languages of lower levels (not mathematics)
- Uses:
  - Vienna Definition Language (VDL) used to define PL/I (Wegner, 1972).
  - Compiler work

Axiomatic Semantics

- Programmers: confirm or prove that a program does what it is supposed to do under al circumstances
- Axiomatic semantics provides a vehicle for developing proofs that a program is "correct".

Axiomatic Semantics

- Example: prove mathematically that the C/C++ function *Max* actually computes as its result the maximum of its two parameter: *a* and *b*.
  - Calling this function one time will obtain an answer for a particular *a* and *b*, such as 8 and 13. But the parameters *a* and *b* define a wide range of integers, so calling it several times with all the different values to prove its correctness would be an infeasible task.

Axiomatic Semantics

- Construct a proof to prove the correctness of a program
  - The meaning of a statement is defined by the result of the logical expression that precedes and follows it.
  - Those logical expressions specifies constraints on program variables.
  - The notation used to describe constraints is *predicate calculus*.

Axiomatic Semantics: assertions

- The logical expressions used in axiomatic semantics are called *assertions.*
- *Precondition:* an assertion immediately preceding a statement that describes the constraints on the program variables at that point.
- *Postcondition:* an assertion immediately following a statement that describes the new constraints on some variables after the execution of the statement.

Axiomatic Semantics: assertions

- Example

```
sum = 2 * x + 1 { sum > 1 }
```

  - Preconditions and postconditions are enclosed in braces
  - Possible preconditions:

```
{ x > 10 }
{ x > 50 }
{ x > 1000 }
{ x > 0 }
```

Axiomatic Semantics: weakest precondition

- It is the least restrictive precondition that will guarantee the validity of the associated postcondition.
- Correctness proof of a program can be constructed if the weakest condition can be computed from the given postcondition.
- Construct preconditions in reverse:
  - From the postcondition of the last statement of the program generate the precondition of the previous statement.
  - This precondition is the postcondition of the previous statement, and so on.

Axiomatic Semantics: weakest precondition

  - The precondition of the first statement states the condition under which the program will compute the desired results.
  - Correct program: If the precondition of the first statement is implied by the input specification of the program.
- The computation of the weakest precondition can be done using:
  - *Axiom:* logical statement that is assumed to be true.
  - *Inference rule:* method of inferring the truth of one assertion on the basis of the values of other assertions.

Axiomatic Semantics: assignment statements

- Let $x=E$ be a general assignment statement and Q its postconditions.
  - Precondition: $P=Q_{x \text{®} E}$
  - P is computed as Q with all instance of x replaced by E
- Example

```
a = b/2-1 {a<10}
```

Weakest precondition: substitute `b/2-1` in the postcondition `{a<10}`

```
b/2-1 < 10
b < 22
```

Axiomatic Semantics: assignment statements

- General notation of a statement: *{P} S {Q}*
- General notation of the assignment statement: $\{Q_{x®E}\}x = E\ \{Q\}$
- More examples:

```
x = 2*y-3 {x>25}    2*y-3 > 25
                    y > 14

x = x+y-3 {x>10}    x+y-3 > 10
                    y > 13-x
```

Axiomatic Semantics: assignment statements

- An assignment with a precondition and a postcondition is a theorem.
  - If the assignment axiom, when applied to the postcondition and the assignment statement, produces the given precondition, the theorem is proved.
- Example:

```
{x > 5} x = x-3 {x>0}
```

Using the assignment axiom on

```
x = x-3 {x>0}
{x > 3}
```

{x > 5} implies {x > 3}

Axiomatic Semantics: sequences

- The weakest precondition cannot be described by an axiom (only with an inference rule)
  - It depends on the particular kinds of statements in the sequence.
- Inference rule:
  - The precondition of the second statement is computed.
  - This is used as the postcondition of the first statement.
  - The precondition of the first element is the precondition of the whole sequence.

Axiomatic Semantics: sequences

- Example:

```
 y = 3*x+1;
 x = y+3;
{x < 10}
```

Precondition of last assignment statement

```
y < 7
```

Used as postcondition of the first statement

```
3*x+1 < 7
x < 2
```

Axiomatic Semantics: selection

- Inference rule:
  - Selection statement must be proven for both when the Boolean control expression is true and when it is false.
  - The obtained precondition should be used in the precondition of both the **then** and the **else** clauses.

Axiomatic Semantics: selection

- Example:

```
 if (x > 0)
     y = y-1
 else y = y+1
 {y > 0}
```

Axiom for assignment on the "then" clause

```
 y = y-1 {y > 0}
 y-1 > 0
 y > 1
```

Same axiom to the "else" clause

```
 y = y+1 {y > 0}
 y+1 > 0
 y > -1
```

But {y > 1} Þ {y > -1}

Precondition of the whole statement: {y > 1}

Axiomatic Semantics: evaluation

- Advantages:
  - Can be very abstract.
  - May be useful in program correctness proofs.
  - Solid theoretical foundations.
- Disadvantages:
  - Predicate transformers are hard to define.
  - Hard to give complete meaning.
  - Does not suggest implementation.
- Uses:
  - Semantics of Pascal.
  - Reasoning about correctness.

Denotational Semantics

- Most rigorous, abstract, and widely known method.
- Based on recursive function theory.
- Originally developed by Scott and Strachery (1970).
- Key idea: define a function that maps a program (a syntactic object) to its meaning (a semantic object).
  - It is difficult to create the objects and mapping functions.

Denotational vs. Operational

- Denotational semantics is similar to high-level operational semantics, except:
  - Machine is gone.
  - Language is mathematics (lambda calculus).
- Differences:
  - In operational semantics, the state changes are defined by coded algorithms for a virtual machine
  - In denotational semantics, they are defined by rigorous mathematical functions.

Denotational Semantics: evaluation

- Advantages:
  - Compact and precise, with solid mathematical foundation.
  - Provides a rigorous way to think about programs.
  - Can be used to prove the correctness of programs.
  - Can be an aid to language design.
- Disadvantages:
  - Requires mathematical sophistication
  - Hard for programmers to use.
- Uses:
  - Semantics for Algol 60
  - Compiler generation and optimization

Summary

- Each form of semantic description has its place:
  - Operational
    - Informal descriptions
    - Compiler work
  - Axiomatic
    - Reasoning about particular properties
    - Proofs of correctness
  - Denotational
    - Formal definitions
    - Probably correct implementations

Chapter 3

Attribute Grammars

Meaning

- What is the semantics or meaning of the expression: 2+3
  - Its value: 5
  - Its type (type checker): int
  - A string (infix-to-postfix translator): + 2 3
- The semantics of a construct can be any quantity or set of quantities associated with the construct.

Attribute Grammars

- Formalism for specifying semantics based on context-free grammars (BNF).
- Used to solve some typical problems:
  - Type checking and type inference
  - Compatibility between procedure definition and call.
- Associate attributes with *terminals* and *nonterminals.*
- Associate semantic functions with *productions.*
  - Used to compute attribute values.

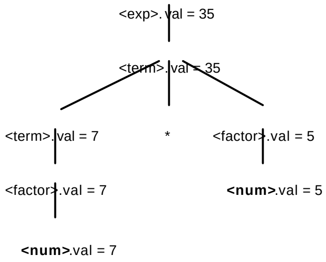
Attributes

- A quantity associated with a construct.
  - X.a for attribute a of X (X is either a *nonterminal* or a *terminal*).
- Attributes have values:
  - Each occurrence of an attribute of an attribute in a parse tree has a value.
- Grammar symbols can have any number of attributes.

Example: Evaluating arithmetic expressions

<exp> ::= <exp> + <term>
<exp> ::= <exp> – <term>
<exp> ::= <term>
<term> ::= <term> * <factor>
<term> ::= <term> div <factor>
<term> ::= <factor>
<factor> ::= (<exp>)
<factor> ::= num

Example: 7*5

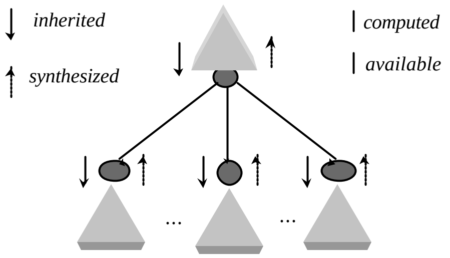

<exp>.val = 35
<term>.val = 35
<term>.val = 7
*
<factor>.val = 5
<factor>.val = 7
<num>.val = 5
<num>.val = 7

- **val** is the value of the digit
- At the root of the parse tree:
  - **<exp>.val** has value **35**
- At the bottom-left:
  - **<num>.val** has value **7**
- Attributes for terminal symbols:
  - come with the symbol
  - the value of the token
- Attribute values for nonterminals:
  - Defined by semantic rules
    - Attached to productions
- *Decorated parse tree*
  - Attributes attached to the nodes

Attributes

- Syntax symbols can return values (sort of output parameters)
  - Digits can return its numeric value
    - digit <?val>
- Nonterminal symbols can have also input attributes.
  - Parameters that are passed from the “calling” production.
    - number <?base, ?val>
      - base: number base (e.g. 10 or 2 or 16)
      - val: returned value of the number

Information Flow



Synthesized Attributes

- The values is computed from the values of attributes of the *children*.
- Pass information up the parse tree (bottom-up propagation).
- *S-attribute grammar* uses only synthesized attributes
- Example:
  - *Value of expressions*
  - *Types of expressions*

Inherited Attributes

- The values is computed from the values of attributes of the *siblings and parent*.
- Pass information down the parse tree (top-down propagation) *or* from left siblings to the right siblings
- Example:
  - *Type information*
  - *Where does a variable occur? LHS or RHS*

Example 1

- Translating decimal numbers between 0 and 99 into their English phrases.

number	*phrase*
0	zero
10	ten
19	nineteen
20	twenty
31	thirty one

  - Translations are based on each digit
    - 31: thirty, the translation of 3 on the left, and one, the translation of 1 on the right.
    - Exceptions:
      - 30 is thirty, not thirty zero
      - 19: is nineteen, not ten nine

Example 1: Syntax

```
<number> ::= <digit>
<number> ::= <digit> <set_digit>
<set_digit> ::= <digit>
<digit> ::= 0|1|2|3|4|5|6|7|8|9
```

```
<N> ::= <D>
<N> ::= <D> <S>
<S> ::= <D>
<D> ::= 0|1|2|3|4|5|6|7|8|9
```

Attribute Occurrences

- Same attribute can be associated with different symbols appearing in the same grammar rule.
- *Attribute occurrence* of a rule *p* is an ordered pair of attributes and natural number *<a,j>* representing the attribute *a* at position *j* in production *p*.
- Two disjoint subsets:
  - *Defined occurrences* for a production:
    - *The information flowing into a node of the parse tree.*
  - *Used occurrences for a production*
    - *The information flowing out a node of the parse tree.*

Used Attribute Occurrences

Rule: *S? AB*

	S	A	B
synthesized	**Syn(S)**	**Syn(A)**	**Syn(B)**
inherited	**In(S)**	**In(A)**	**In(B)**

- Set of inherited attributes of all the grammar symbols on the LHS plus the set of synthesized attributes of the RHS.

Defined Attribute Occurrences

Rule: *S? AB*

	S	A	B
synthesized	**Syn(S)**	**Syn(A)**	**Syn(B)**
inherited	**In(S)**	**In(A)**	**In(B)**

- Set of synthesized attributes of all the grammar symbols on the LHS plus the set of inherited attributes of the RHS.

Semantic Function

- Define a semantic function for every defined occurrence in terms of the values of used occurrences.

	Defined	Used
Rule 1	…	…
Rule 2	…	…

Function definitions

Example 1: Semantics

<N> ::= <D>	N.trans := spell(D.val)
<N> ::= <D> <S>	S.in ::= D.val N.trans ::= S.trans
<S> ::= <D>	S.val := **if** D.val = 0 **then** *decade*(S.in) **else if** S.in ≤ 1 **then** *spell*(10*S.in +D.val) **else** *decade*(P.in) || *spell*(D.val)
<D> ::= 0	<D>.val := 0
…	
<D> ::= 9	<D>.val := 9

Functions *spell* and *decade:*

spell(1) = one, *spell*(2) = two, …, *spell*(19) = nineteen

decade(0) = zero, *decade*(1) = ten, …, *decade*(9) = ninety

Example 2: Syntax

Decimal value of a binary number

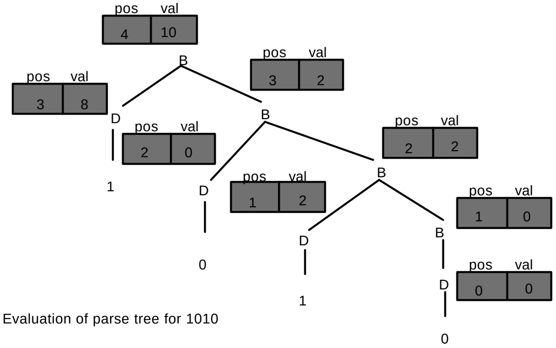
<binary> ::= <digit>
<binary> ::= <digit> <binary>
<digit> ::= 0
<digit> ::= 1

<B> ::= <D>
<B> ::= <D> <B>
<D> ::= 0
<D> ::= 1

Example 2: Semantics

<B> ::= <D>	B.pos := 1 B.val := D.val D.pow := 0
<B_1> ::= <D> <B_2>	B_1.pos := B_2.pos + 1 B_1.val := B_2.val + D.val D.pow := B_2.pos
<D> ::= 0	D.val := 0
<D> ::= 1	D.val := $2^{D.pow}$

Example 2: Sample Parse Tree



Evaluation of parse tree for 1010

Example 3: Syntax

Simple Assignment Statements

<assign> ::= <var> = <expr>
<expr> ::= <var> + < var>
<expr> ::= <var>
<var> ::= X | Y | Z

<A> ::= <V> = <E>
<E> ::= <V> + <V>
<E> ::= <V>
<V> ::= X | Y | Z

Example 3: Semantics

<A> ::= <V> = <E>	E.exp := V.act
<E> ::= <V> + <V>	E.act = if (V_1.act = int) and V_2.act := int) then int else real
<E> ::= <V>	E.act := E.exp
<V> ::= X | Y | Z	V.act = …

Variables can be either real or integer.
Both sides of an assignment different: type = real
Same type on both sides of an assignment

Attribute Grammars: Summary

- An attribute grammar is a context-free grammar with two disjoint sets of attributes (inherited and synthesized) and semantic functions for all defined attribute occurrences.

Attribute Grammar: Process

1. EBNF
2. Attributes
   - Identify the parameters of the syntax symbols.
     - Output attributes (synthesized) yield results.
     - Input attributes (inherited) provide context.
3. Semantic functions

Chapter 3

Operational Semantics

Dynamic Semantics

- Semantics of a programming language is the definition of the *meaning* of any program that is syntactically valid.
- Intuitive idea of programming meaning: “whatever happens in a (real or model) computer when the program is executed.”
  - A precise characterization of this idea is called *operational semantics.*

Operational Semantics: advantages and disadvantages

- Operational Semantics
  - Advantage of representing program meaning directly in the code of a real (or simulated) machine.
  - Potential weakness, since the definition of semantics is confined to a particular architecture (either real or abstract).
    - Virtual machine also needs a semantic description, which adds complexity and can lead to circular definitions.

Operational Semantics

- Provides a definition of program meaning by simulating the program's behavior on a machine model that has a very simple (through not necessarily realistic) instruction set and memory organization.
- Definition of the virtual computer can be described using an existing programming language or a virtual computer (idealized computer).
- Change in the state of the machine (memory, registers, etc) defines the meaning of the statement.

Process

- The process:
  - Identify a virtual machine (an idealized computer).
  - Build a translator (translates source code to the machine code of an idealized computer).
  - Build a simulator for the idealized computer.
- Operational semantics is sometimes called *transformational semantics*, if an existing programming language is used in place of the virtual machine.

Example

Pascal statement

```
for i := x to y do
   begin
      ...
   end
```

Operational Semantics (lower level)

```
movi, r1
movy, r2
jmpifless(r2,r1,out)
...
```

Operational Semantics

```
           i := x
loop: if i > y goto out
           ...
           i := i + 1
           goto loop
out: ...
```

Notation

- State of a program σ:
  - A set of pairs <v,val> that represent all active variables and their current assigned values at some stage during the program’s execution.
    - $\sigma = \{ <x,1>, <y,2>, <z,3> \}$
    - After $y = 2 * z + 3$ $\sigma = \{ <x,1>, <y,9>, <z,3> \}$
    - After $w = 4$ $\sigma = \{ <x,1>, <y,9>, <z,3>, <w,4> \}$
- State transformation of these type of assignments can be represented by a function called *overriding union U*
  - $\sigma_1 = \{ <x,1>, <y,2>, <z,3> \}$
  - $\sigma_2 = \{ <y,9>, <w,4> \}$
  - $\sigma_1 \text{ U } \sigma_2 = \{ <x,1>, <y,9>, <z,3>, <w,4> \}$

Notation

- *Execution rule*:

$$\frac{\textit{premise}}{\textit{conclusion}}$$

  - "If the *premise* is true, then the *conclusion* is true"

Examples

- Addition of two expressions

$$\frac{s(e_1) \text{ Þ } v_1 \quad s(e_1) \text{ Þ } v_1}{s(e_1 + e_2) \text{ Þ } v_1 + v_2}$$

- Assignment statement (*s.target = s.source)*

$$\frac{s(s.source) \text{ Þ } v}{s(s.target = s.source) \text{ Þ } s \text{ U } \{ <s.target,v> \}}$$

  - Suppose: assignment x = x +1, current state x=5

$$\frac{\dfrac{s(x) \text{ Þ } 5 \quad s(1) \text{ Þ } 1}{s(x+1) \text{ Þ } 6}}{s(x = x+1) \text{ Þ } \{\ldots, <x,5>, \ldots\} \text{ U } \{ <x,6> \}}$$

Examples

- Conditionals (s = if (s.text) s.then else s.else)

$$\frac{s(s.test) \text{ Þ } true \quad s(s.then) \text{ Þ } s_1}{s(if(s.test)s.then\ else\ s.else) \text{ Þ } s_1}$$

$$\frac{s(s.test) \text{ Þ } false \quad s(s.else) \text{ Þ } s_1}{s(if(s.test)s.then\ else\ s.else) \text{ Þ } s_1}$$

Examples

- Loops (s = while (s.test) s.body)

$$\frac{s(s.test) \text{ Þ } true \quad s\ (s.body) \text{ Þ } s_1 \quad s_1(while(s.test)s.body) \text{ Þ } s_1}{s(while\ (s.text)\ s.body) \text{ Þ } s_1}$$

$$\frac{s(s.test) \text{ Þ } false}{s(\ while\ (s.text)\ s.body) \text{ Þ } s}$$

Evaluation

- Advantages:
  - May be simple, intuitive for small examples/
  - Good if used informally.
  - Useful for implementation.
- Disadvantages:
  - Very complex for large programs.
  - Depends on programming languages of lower levels (not mathematics)
- Uses:
  - Vienna Definition Language (VDL) used to define PL/I (Wegner, 1972).
  - Compiler work

Chapter 3

Axiomatic Semantics

Dynamic Semantics

- Another way to view programming meaning is to start with a formal specification of what a program is supposed to do, and then rigorously prove that the program does that by using a systematic series of logical steps.
  - This approach evokes the idea of *axiomatic semantics*.

Axiomatic Semantics

- Programmers: confirm or prove that a program does what it is supposed to do under al circumstances
- Axiomatic semantics provides a vehicle for developing proofs that a program is “correct”.

Axiomatic Semantics

- Example: prove mathematically that the C/C++ function *Max* actually computes as its result the maximum of its two parameter: *a* and *b*.
  - Calling this function one time will obtain an answer for a particular *a* and *b*, such as 8 and 13. But the parameters *a* and *b* define a wide range of integers, so calling it several times with all the different values to prove its correctness would be an infeasible task.

Assertions

- The logical expressions used in axiomatic semantics are called *assertions.*
- *Precondition:* an assertion immediately preceding a statement that describes the constraints on the program variables at that point.
- *Postcondition:* an assertion immediately following a statement that describes the new constraints on some variables after the execution of the statement.

Assertions

- Example

```
sum = 2 * x + 1 { sum > 1 }
```

  - Preconditions and postconditions are enclosed in braces
  - Possible preconditions:

```
{ x > 10 }
{ x > 50 }
{ x > 1000 }
{ x > 0 }
```

Weakest Precondition

- It is the least restrictive precondition that will guarantee the validity of the associated postcondition.
- Correctness proof of a program can be constructed if the weakest condition can be computed from the given postcondition.
- Construct preconditions in reverse:
  - From the postcondition of the last statement of the program generate the precondition of the previous statement.
  - This precondition is the postcondition of the previous statement, and so on.

Weakest Precondition

  - The precondition of the first statement states the condition under which the program will compute the desired results.
  - Correct program: If the precondition of the first statement is implied by the input specification of the program.
- The computation of the weakest precondition can be done using:
  - *Axiom:* logical statement that is assumed to be true.
  - *Inference rule:* method of inferring the truth of one assertion on the basis of the values of other assertions.

Assignment Statements

- Let *x=E* be a general assignment statement and *Q* its postconditions.
  - Precondition: $P=Q_{x\rightarrow E}$
  - *P* is computed as *Q* with all instance of *x* replaced by *E*
- Example

    a = b/2-1 {a<10}

Weakest precondition: substitute *b/2-1* in the postcondition *{a<10}*

    b/2-1 < 10

    b < 22

Assignment Statements: examples

- General notation of a statement: *{P} S {Q}*
- More examples:
  - *x = 4*y+5 { x>13 }*
  - *X = y-3*6 { x>-5 }*
  - *X = 2*y+3*x { x>10}*

Assignment Statements

- An assignment with a precondition and a postcondition is a theorem.
  - If the assignment axiom, when applied to the postcondition and the assignment statement, produces the given precondition, the theorem is proved.
- Example:

    {x > 5} x = x-3 {x>0}

Using the assignment axiom on

    x = x-3 {x>0}

    {x > 3}

{x > 5} implies {x > 3}

Sequences

- The weakest precondition for a sequence cannot be described by an axiom (only with an inference rule)
  - It depends on the particular kinds of statements in the sequence.
- Inference rule:
  - The precondition of the second statement is computed.
  - This is used as the postcondition of the first statement.
  - The precondition of the first element is the precondition of the whole sequence.

Sequences: examples

- Example:

    *y = 3*x+1;*

    x = y+3;

    {x < 10}

Precondition of last assignment statement

    y < 7

Used as postcondition of the first statement

    *3*x+1 < 7*

    x < 2

- Other example:

    a = 3*(2*b+a);

    b = 2*a -1

    { b > 5 }

Selection

- Inference rule:
  - Selection statement must be proven for both when the Boolean control expression is true and when it is false.
  - The obtained precondition should be used in the precondition of both the **then** and the **else** clauses.

Selection: example

- Example:

```
 if (x > 0)
    y = y-1
else y = y+1
{y > 0}
```

Axiom for assignment on the "then" clause

```
 y = y-1 {y > 0}
y-1 > 0
y > 1
```

Same axiom to the "else" clause

```
 y = y+1 {y > 0}
y+1 > 0
y > -1
```

But {y > 1} Þ {y > -1}

Precondition of the whole statement: {y > 1}

Evaluation

- Advantages:
  - Can be very abstract.
  - May be useful in program correctness proofs.
  - Solid theoretical foundations.
- Disadvantages:
  - Predicate transformers are hard to define.
  - Hard to give complete meaning.
  - Does not suggest implementation.
- Uses:
  - Semantics of Pascal.
  - Reasoning about correctness.

Chapter 3

Denotational Semantics

Dynamic Semantics

- A third way to view the semantics of a programming language is to define the meaning of each type of statement that occurs in the (abstract) syntax as a state-transforming mathematical function.
  - The meaning of a program can be expressed as a collection of functions operating on the program state.
  - This approach is called *denotational semantics.*

Denotational Semantics

- Most rigorous, abstract, and widely known method.
- Based on recursive function theory.
- Originally developed by Scott and Strachery (1970).
- Key idea: define a function that maps a program (a syntactic object) to its meaning (a semantic object).
  - It is difficult to create the objects and mapping functions.

Denotational vs. Operational

- Denotational semantics is similar to high-level operational semantics, except:
  - Machine is gone.
  - Language is mathematics (lambda calculus).
- Differences:
  - In operational semantics, the state changes are defined by coded algorithms for a virtual machine
  - In denotational semantics, they are defined by rigorous mathematical functions.

Denotational Semantics: evaluation

- Advantages:
  - Compact and precise, with solid mathematical foundation.
  - Provides a rigorous way to think about programs.
  - Can be used to prove the correctness of programs.
  - Can be an aid to language design.
- Disadvantages:
  - Requires mathematical sophistication
  - Hard for programmers to use.
- Uses:
  - Semantics for Algol 60
  - Compiler generation and optimization

Summary

- Each form of semantic description has its place:
  - Operational
    - Informal descriptions
    - Compiler work
  - Axiomatic
    - Reasoning about particular properties
    - Proofs of correctness
  - Denotational
    - Formal definitions
    - Probably correct implementations