From Logic Sentences to Clause Form to Horn Clauses

The first operation is to ***simplify*** the formula so that they contain only the $\forall$, $\exists$, $\wedge$, $\vee$ and $\neg$ connectives. This is done by using the implication identity.

Simplify	
Formula	Rewrites to
$P \supset Q$	$\neg P \vee Q$

The second operation is to ***move negations in*** so that they apply to only atoms. This is done by using known logical identities (including De Morgan's laws).

Move negations in	
Formula	Rewrites to
$\neg\forall x\, P$	$\exists x\, \neg P$
$\neg\exists x\, P$	$\forall x\, \neg P$
$\neg (P \wedge Q)$	$\neg P \vee \neg Q$
$\neg (P \vee Q)$	$\neg P \wedge \neg Q$
$\neg\neg P$	P

The third operation is to ***Skolemize*** to remove existential quantifiers. This step replaces existentially quantified variables by *Skolem functions*. For example, convert $(\exists x)P(x)$ to $P(c)$ where c is a brand new constant symbol that is not used in any other sentence (c is called a *Skolem constant*). More generally, if the existential quantifier is within the scope of a universal quantified variable, then introduce a Skolem function that depends on the universally quantified variable. For example, $\forall x\, \exists y\, P(x,y)$ is converted to $\forall x\, P(x, f(x))$. f is called a *Skolem function*, and must be a brand new function name that does not occur in any other part of the logic sentence.
Example: $(\forall x)(\exists y)loves(x,y)$ is converted to $(\forall x)loves(x,f(x))$ where in this case $f(x)$ specifies the person that x loves. If we knew that everyone loved their mother, then f could stand for the mother-of function.

Skolemize	
Formula	Rewrites to
$\exists x\, P(x)$	$P(c)$
$\forall x\, \exists x\, P(x,y)$	$\forall x\, P(x,f(x))$

The fourth operation is to *remove universal quantifier*. First it is necessary to standardize variables: rename all variables so that each quantifier has its own unique variable name. For example, convert $\forall x\, P(x)$ to $\forall y\, P(y)$ if there is another place where variable x is already used. Remove universal quantification symbols by first moving them all to the left end and making the scope of each the entire sentence, and then just dropping the "prefix" part. For example, convert $\forall x\, P(x)$ to $P(x)$.

The fifth operation is to *distribute disjunctions* so that the formula is a set of conjunctions of disjunctions.

Distribute disjunctions	
Formula	Rewrites to
$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
$(Q \wedge R) \vee P$	$(Q \vee P) \wedge (R \vee P)$

The sixth operation is to *convert to Clause Normal Form*. This is done by removing the conjunctions, to form a set of clauses. Each clause is just a disjunction of negated and un-negated literals.

Convert to CNF	
Formula	Rewrites to
$P_1 \wedge ... \wedge P_n$	$\{P_1,\ldots,P_n\}$

The last operation is to *convert to Horn Clauses*. This operation is not always possible. Horn clauses are clauses in normal form that have one or zero positive literals. The conversion from a clause in normal form with one or zero positive literals to a Horn clause is done by using the implication property.

Simplify	
Formula	Rewrites to
$\neg P \vee Q$	$P \supset Q$

Example 1
Predicate ∀x (¬literate(x) ⊃ (¬writes(x) ∧ ¬∃y(reads(x,y) ∧ book(y)))) Simplify ∀x (literate(x) ∨ (¬writes(x) ∧ ¬∃y(reads(x,y) ∧ book(y)))) Move negations in ∀x (literate(x) ∨ (¬writes(x) ∧ ∀y(¬(reads(x,y) ∧ book(y))))) ∀x (literate(x) ∨ (¬writes(x) ∧ ∀y(¬reads(x,y) ∨ ¬book(y)))) No Skolemize (there are no existential quantifiers) Remove universal quantifier ∀x ∀y (literate(x) ∨ (¬writes(x) ∧ (¬reads(x,y) ∨ ¬book(y)))) literate(x) ∨ (¬writes(x) ∧ (¬reads(x,y) ∨ ¬book(y))) Distribute disjunctions (literate(x) ∨ ¬writes(x)) ∧ (literate(x) ∨ ¬reads(x,y) ∨ ¬book(y)) (¬writes(x) ∨ literate(x)) ∧ (¬reads(x,y) ∨ ¬book(y) ∨ literate(x)) Convert to Clause Normal Form ¬writes(x) ∨ literate(x) ¬reads(x,y) ∨ ¬book(y) ∨ literate(x)

Convert to Horn Clauses

$writes(x) \supset literate(x)$

$reads(x,y) \wedge book(y) \supset literate(x)$

Example 2

Predicate

$\forall x\ (literate(x) \supset reads(x) \vee write(x))$

Simplify

$\forall x\ (\neg literate(x) \vee reads(x) \vee write(x))$

The negations are already in

$\forall x\ (\neg literate(x) \vee reads(x) \vee write(x))$

No Skolemize (there are no existential quantifiers)

Remove universal quantifier

$\neg literate(x) \vee reads(x) \vee write(x)$

No disjunctions

It is already a Clause Normal Form

$\neg literate(x) \vee reads(x) \vee write(x)$

It is not possible to convert to Horn Clauses because there are two positive literals (reads(x) and write(x)).