CMPT 371: Homework 4

Due: April 4, 2007

1. (a)

1	0	1	1	1
0	1	1	0	0
0	0	0	0	0
1	1	1	1	0
0	0	1	0	

(b)

1	0	1	1	1
0	1	1	0	0
0	0	0	0	0
1	1	1	1	0
0	0	1	0	

(c) The following corrupted message could have come from two different original messages by making 2 errors in each:

0	1	1	1	1
0	1	1	0	0
0	0	0	0	0
1	1	1	1	0
0	0	1	0	

The two original messages are

1	0	1	1	1
0	1	1	0	0
0	0	0	0	0
1	1	1	1	0
0	0	1	0	

and

0	1	1	1	1
0	1	1	0	0
0	0	0	0	0
1	1	1	1	0
1	1	1	0	

2. This question is about Cyclic Redundancy Checks (CRCs).

(a)
$$\frac{x^7 + x^5 + x^3 + x}{x^3 + x + 1} = x^4 + x + 1,$$
with remainder $x^2 + x + 1$. Therefore, the CRC is 111.

(b) Notice that

$$\frac{(x^7+x^5+x^3+x)+(x^3+x+1)}{x^3+x+1} = \frac{x^7+x^5+x^3+x}{x^3+x+1} + \frac{x^3+x+1}{x^3+x+1} = \frac{x^7+x^5+x^3+x}{x^3+x+1} + 1,$$

where all additions are done modulo 2. Therefore, the polynomial $(x^7+x^5+x^3+x)+(x^3+x+1)$ has the same remainder when divided by (x^3+x+1) as just $(x^7+x^5+x^3+x)$ does. So, M' corresponds to the polynomial $(x^7+x^5+x^3+x)+(x^3+x+1) = x^7+x^5+1$. As a bit string, it is 10100001, which has distance 3 from 10101010.

3. Consider a channel of rate R bits/second that is shared by N nodes. Assume that $\sqrt{N}$ nodes have data to send.

(a) We would want each node to have throughput $R/\sqrt{N}$ because there are only $\sqrt{N}$ nodes sending data. In TDM, each node must have its turn regardless of whether it has data to send, so each node has throughput R/N.

(b) To pass the token all the way around, it takes $\sqrt{N}T + N\epsilon$ seconds. Each node with data is broadcasting for T seconds of this time. Therefore, each node has throughput

$$R \times \frac{T}{\sqrt{N}T + N\epsilon}.$$

As ϵ tends to 0, this approaches $R/\sqrt{N}$; just what we would hope.

(c) The time to go through 100 cycles is now $100(\sqrt{N}T + N\epsilon) + K$ seconds. Each node sends for $100T$ seconds of this time. Therefore, the throughput of each node is

$$R \times \frac{100T}{100(\sqrt{N}T + N\epsilon) + K}.$$

(d) Here we can ignore all the nodes that don't have data. Consider one particular node that does have data. We want to know what fraction of the time this node is sending. Equivalently, for a given point in time, we want to know the probability that our node is sending. This is (just using the calculation in the book where we replace N by $\sqrt{N}$):

$$p(1-p)^{2\sqrt{N}}.$$

If $p = 1/\sqrt{N}$ and N gets large, $(1-p)^{\sqrt{N}}$ approaches $1/e$. Therefore, our node is sending for a $1/e^2\sqrt{N}$ fraction of the time, so its throughput is $R/e^2\sqrt{N}$. This is similar to the throughput in (b); it's better if ϵ is big, but worse if ϵ is small.