# Design Theory for Relational Databases

- Functional Dependencies
- Decompositions
- Normal Forms: BCNF, Third Normal Form
- Introduction to Multivalued Dependencies

# Design Theory for Relational Databases

- A poor choice of a relational database schema can lead to redundancy and related anomalies.
- We've seen the E/R approach for overall database schema design.
- Here we want to look at relation schemas and
  - Consider problems that might arise with a poor choice of schema,
  - Evaluate a schema with regards to redundancy and other anomalies, and
  - Determine how to come up with a better design by decomposing a relational schema.
- Key notion: functional dependency.

# Functional Dependencies

- Functional dependencies generalize the notion of a *key* of a relation.
- Write a FD as $X \rightarrow Y$ where X and Y are sets of attributes
- $X \rightarrow Y$ is an assertion about a relation $R$ that *whenever two tuples of $R$ agree on all the attributes of X, then they must also agree on all attributes in set Y.*
  - Say "$X \rightarrow Y$ holds in $R$."
  - Convention: …, $X$, $Y$, $Z$ represent sets of attributes;

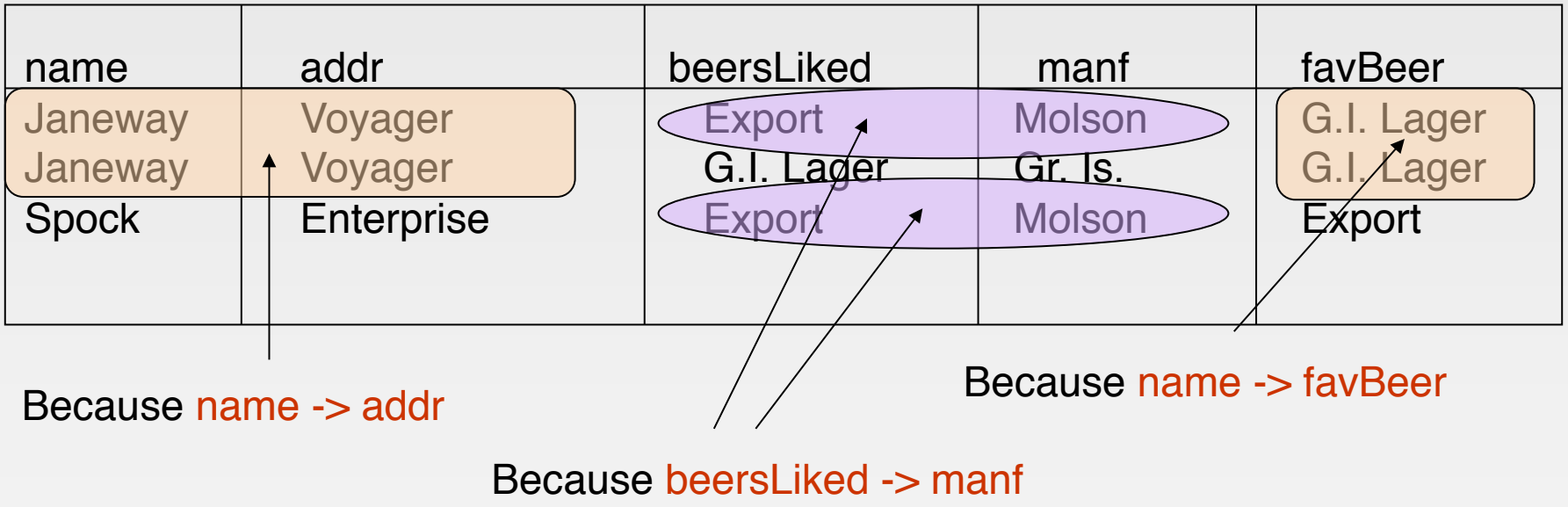
    $A$, $B$, $C$,… represent single attributes.
  - Convention: No set notation for sets of attributes; use $ABC$, rather than $\{A,B,C\}$.

# Example: FD's

Customers(name, addr, beersLiked, manf, favBeer)

- Reasonable FD's to assert:
  1. name -> addr favBeer
     - Note this FD is the same as name -> addr and name -> favBeer.
  2. beersLiked -> manf

# Example: Possible Data



# Splitting Right Sides of FD's

- $X \rightarrow A_1 A_2 \ldots A_n$ holds for $R$ exactly when each of $X \rightarrow A_1$, $X \rightarrow A_2$,…, $X \rightarrow A_n$ hold for $R$.
- Example: $A \rightarrow BC$ is equivalent to $A \rightarrow B$ and $A \rightarrow C$.
- There is no splitting rule for left sides.
- We'll generally express FD's with singleton right sides.

# Keys of Relations

- Let *K* be a set of attributes (possibly singleton) in a relation *R*
- *K* is a *superkey* for relation *R* if *K* functionally determines all attributes of *R*.
- *K* is a *key* for *R* if *K* is a superkey, but no proper subset of *K* is a superkey.
  - Also called a candidate key
- A *primary key* is a candidate key that has been selected as the means of identifying tuples in a relation.

# Example: Superkey

Customers(name, addr, beersLiked, manf, favBeer)

- {name, beersLiked} is a superkey because together these attributes determine all the other attributes.
  - name -> addr favBeer
  - beersLiked -> manf

# Example: Key

- {name, beersLiked} is a key because neither {name} nor {beersLiked} is a superkey.
  - name doesn't -> manf;  beersLiked doesn't -> addr.
- There are no other keys, but lots of superkeys.
  - Any superset of {name, beersLiked} is a superkey.

# Where Do Keys Come From?

1. Just assert a key *K*
   - E.g. student number
   - Have FD's *K -> A* for all attributes *A.*
2. Determine FD's and deduce the keys by systematic exploration.

# More FD's From "Physics"

- Example: "no two courses can meet in the same room at the same time" tells us: hour room -> course.
  - I.e. commonsense constraints

# Inferring FD's

- We are given FD's

  $X_1 \rightarrow A_1,\ X_2 \rightarrow A_2,\ \ldots,\ X_n \rightarrow A_n$,

  and we want to know whether an FD $Y \rightarrow B$ must hold in any relation that satisfies the given FD's.

  - Example: If $A \rightarrow B$ and $B \rightarrow C$ hold, surely $A \rightarrow C$ holds, even if we don't say so.
- Important for design of good relation schemas.

# Inference Test

- To test if $Y \rightarrow B$ holds, given a set of FDs, start by assuming that two tuples agree in all attributes of $Y$.

$\leftarrow Y \rightarrow$

$a_1 a_2 a_3 b_1 b_2 \ldots$

$a_1 a_2 a_3 c_1 c_2 \ldots$

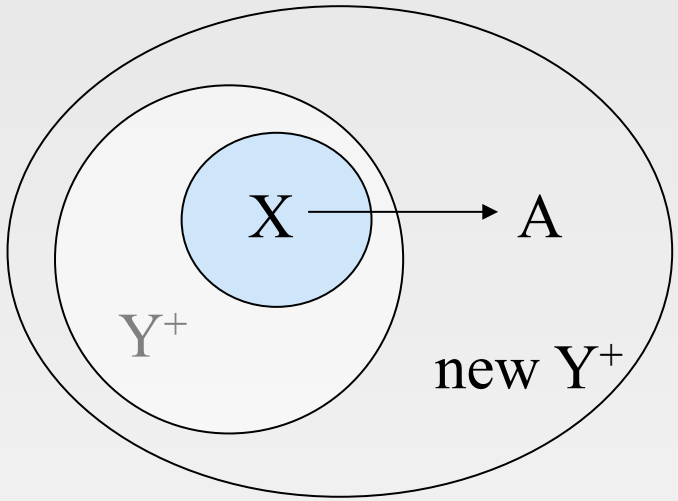
- Use the given FD's to infer that these tuples must also agree in certain other attributes.
  - If B is one of these attributes, then $Y \rightarrow B$ is true.
  - Otherwise, the two tuples, with any forced equalities, form a two-tuple relation that proves $Y \rightarrow B$ does not follow from the given FD's.

# Closure Test

- An easier way to test is to compute the *closure* of $Y$, denoted $Y^+$.
- Basis: $Y^+ = Y$.
- Induction: Look for a FD whose left side $X$ is a subset of the current $Y^+$.
  - If the FD is $X \rightarrow A$, add $A$ to $Y^+$.

# Diagramatically:

Given $Y^+$ and X -> A:



# Finding All Implied FD's

- Sometimes, for a relation R with a set of FD's, we want to find those FD's that hold in subrelations of R.
- Motivation: "normalization," the process where we break a relation schema into two or more schemas for better performance
  - More on normalization later...
- Example: *ABCD* with FD's *AB ->C*, *C ->D*, and *D ->A*.
  - Decide to decompose into *ABC*, *AD*.
  - Ask: what FD's hold in *ABC* ?
  - Answer: not only *AB ->C*, but also *C ->A* !

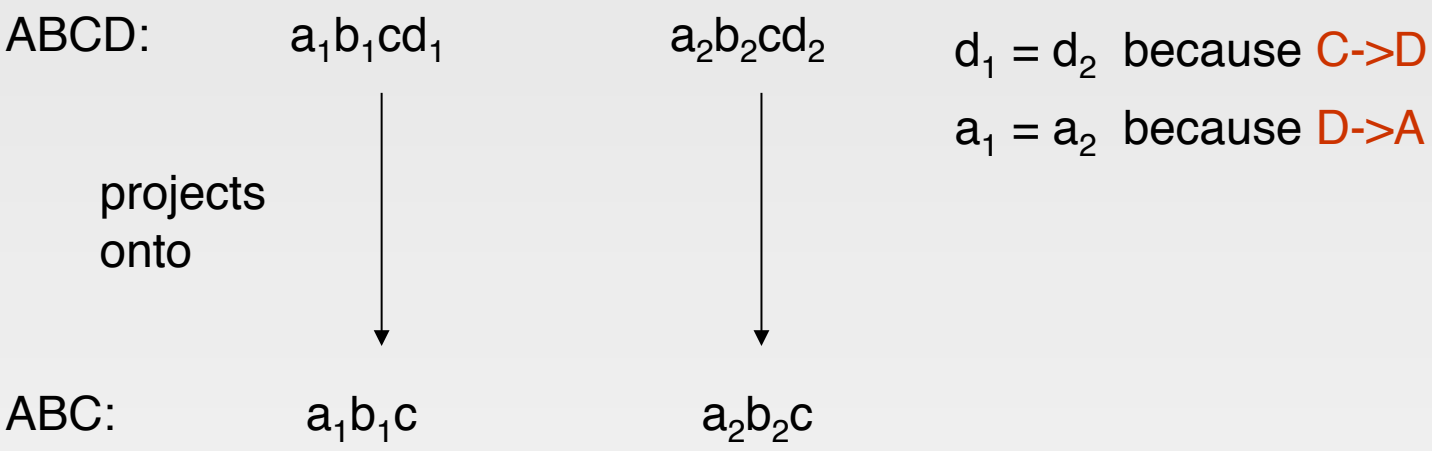
# Why?

ABCD: $a_1b_1cd_1$ $a_2b_2cd_2$

$d_1 = d_2$ because C->D

$a_1 = a_2$ because D->A

# Why?




$d_1 = d_2$ because C->D

$a_1 = a_2$ because D->A

Thus, tuples in the projection with equal C's have equal A's. Hence *C -> A*.

# Find FDs of a Projected Schema: Basic Idea

1. Start with given FD's and find *all nontrivial* FD's that follow from the given FD's.
   - Nontrivial = right side not contained in the left.
2. Select those FD's that involve only attributes of the projected schema.

# Simple, Exponential Algorithm

1. For each set of attributes $X$, compute $X^+$.
2. Add $X \rightarrow A$ for all $A$ in $X^+ - X$.
3. However, drop $XY \rightarrow A$ whenever we discover $X \rightarrow A$.
   - Because $XY \rightarrow A$ follows from $X \rightarrow A$.
4. Finally, use only FD's involving projected attributes.

# A Few Tricks

- Trivially, there's no need to compute the closure of the empty set or of the set of all attributes.
- If we find $X^+$ = all attributes, the closure of any superset of $X$ is also the set of all attributes.
  - So if $X^+$ = all attributes, you don't need to consider any supersets of X.

# Example: Projecting FD's

- *ABC* with FD's *A* ->*B* and *B* ->*C*.

  Ask, what FDs hold on *AC*?
- Compute the closure.
  - $A^+ = ABC$ ; yields *A* ->*B*, *A* ->*C*.
    - We do not need to compute $(AB)^+$ or $(AC)^+$.
  - $B^+ = BC$ ; yields *B* ->*C*.
  - $C^+ = C$ ; yields nothing new.
  - $(BC)^+ = BC$ ; yields nothing new.
  - Resulting FD's: *A* ->*B*, *A* ->*C*, and *B* ->*C*.
- Projection onto *AC* : *A* ->*C*.
  - This is the only FD that involves {*A*,*C* }.

# Relational Schema Design

- We can use FD's to help us design a good relational schema, given an existing database schema
- Goal of relational schema design is to avoid anomalies and redundancy.
  - *Update anomaly*: one occurrence of a fact is changed, but not all occurrences.
  - *Deletion anomaly*: valid fact is lost when a tuple is deleted.
- Overall idea: Ensure that relations are in some *normal form*, which will guarantee that the relation has certain (good) properties.
- We'll look at:
  - Boyce-Codd Normal Form (BCNF)
  - 3rd Normal Form
  - And briefly look at 4th Normal Form

# Example of Bad Design

Customers(name, addr, beersLiked, manf, favBeer)

| name | addr | beersLiked | manf | favBeer |
|---|---|---|---|---|
| Janeway | Voyager | Export | Molson | G.I. Lager |
| Janeway | ??? | G.I. Lager | Gr. Is. | ??? |
| Spock | Enterprise | Export | ??? | Export |

Data is redundant, because each of the ???'s can be figured out by using the FD's name -> addr favBeer and beersLiked -> manf.

# This Bad Design Also Exhibits Anomalies

| name | addr | beersLiked | manf | favBeer |
|---|---|---|---|---|
| Janeway | Voyager | Export | Molson | G.I. Lager |
| Janeway | Voyager | G.I. Lager | Gr. Is. | G.I. Lager |
| Spock | Enterprise | Export | Molson | Export |

- Update anomaly: If Janeway is transferred to Intrepid, we have to remember to change each of her tuples.
- Deletion anomaly: If nobody likes Export, we lose track of the fact that Molson manufactures Export.

# Boyce-Codd Normal Form

- We say a relation *R* is in *BCNF* if whenever *X ->Y* is a nontrivial FD that holds in *R*, *X* is a superkey.
- Remember:
  - *nontrivial* means *Y* is not contained in *X*.
  - *superkey* is any superset of a key (not necessarily a proper superset).

# Example

Customers(name, addr, beersLiked, manf, favBeer)

FD's: name->addr favBeer,  beersLiked->manf

- The only key is {name, beersLiked}.
- In each FD, the left side is *not* a superkey.
- Either of these FD's shows that *Customers* is not in BCNF

# Another Example

Beers(name, manf, manfAddr)

FD's: name->manf,  manf->manfAddr

- The only key is {name} .
- name->manf does not violate BCNF, but manf->manfAddr does.

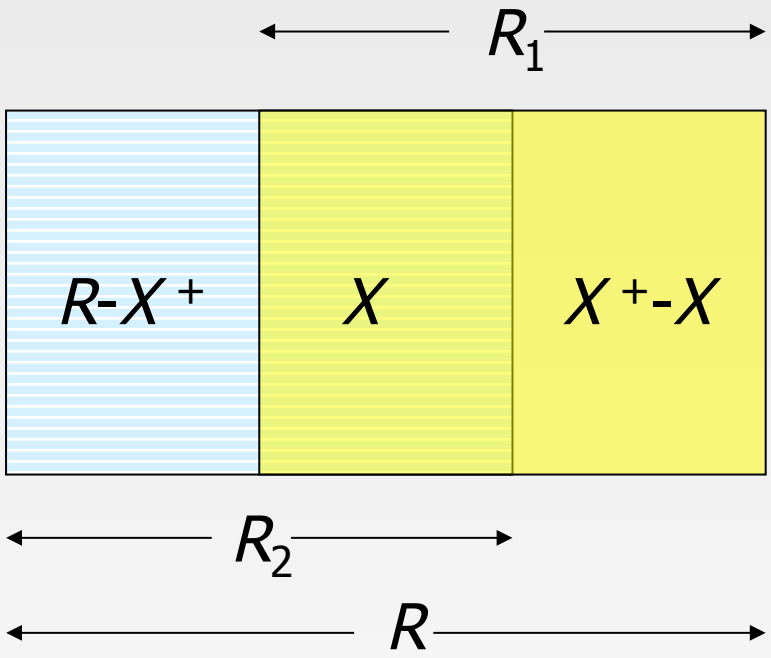
# Decomposition into BCNF

- Goal: For relation R not in BCNF, decompose R into subrelations that are in BCNF.
- Given: Relation *R* with FD's *F*.
- Look among the given FD's for a BCNF violation *X* ->*Y*.
  - If any FD following from *F* violates BCNF, then there will surely be an FD in *F* itself that violates BCNF.
- Compute $X^+$.
  - We won't get all attributes, since X isn't a superkey.

# Decompose *R* using *X* -> *Y*

- *Replace* *R* by relations with schemas:
  1. $R_1 = X^+$.
  2. $R_2 = R - (X^+ - X)$.
- *Project* the given FD's *F* onto the two new relations.
  - Recall that this requires finding the implicit FD's.

# Decomposition Picture



# Example: BCNF Decomposition

Customers(name, addr, beersLiked, manf, favBeer)

*F* = name->addr, name -> favBeer, beersLiked->manf

- Pick BCNF violation name->addr.
- Close the left side: $\{name\}^+ = \{name, addr, favBeer\}$.
- Decomposed relations:
  1. Customers1(name, addr, favBeer)
  2. Customers2(name, beersLiked, manf)

# Example -- Continued

- We are not done.
  - We need to check Customers1 and Customers2 for BCNF violations.
- Projecting FD's is easy here.
- For Customers1(name, addr, favBeer), relevant FD's are name->addr and name->favBeer.
  - Thus, {name} is the only key and Customers1 is in BCNF.

# Example -- Continued

- For Customers2(name, beersLiked, manf), the only FD is beersLiked->manf, and the only key is {name, beersLiked}.
  - Violation of BCNF.
- beersLiked$^+$ = {beersLiked, manf}, so we decompose *Customers2* into:
  1. Customers3(beersLiked, manf)
  2. Customers4(name, beersLiked)

# Example -- Concluded

- The resulting decomposition of *Customers* :
  1. Customers1(name, addr, favBeer)
  2. Customers3(beersLiked, manf)
  3. Customers4(name, beersLiked)
- Notice:
  - *Customers1* tells us about customers,
  - *Customers3* tells us about beers, and
  - *Customers4* tells us the relationship between customers and the beers they like.

# Third Normal Form -- Motivation

- There is one configuration of FD's that causes trouble when we decompose.
- *AB ->C* and *C ->B*.
  - Example: *A* = street address, *B* = city, *C* = postal code.
- There are two keys, {*A,B* } and {*A,C* }.
- *C ->B* is a BCNF violation, so to get BCNF we decompose into *AC*, *BC*.

# We Cannot Enforce FD's

- The problem is that if we use *AC* and *BC* as our database schema, we cannot enforce the FD *AB ->C* by checking FD's in these decomposed relations.
- Example with *A* = street, *B* = city, and *C* = postal code on the next slide.

# An Unenforceable FD

| street | p.c. |
|---|---|
| 545 Tech Sq. | 02138 |
| 545 Tech Sq. | 02139 |

| city | p.c. |
|---|---|
| Cambridge | 02138 |
| Cambridge | 02139 |

Join tuples with equal postal codes.

| street | city | p.c. |
|---|---|---|
| 545 Tech Sq. | Cambridge | 02138 |
| 545 Tech Sq. | Cambridge | 02139 |

Although no FD's were violated in the decomposed relations,
FD street city -> p.c. is violated by the database as a whole.

# 3NF Let's Us Avoid This Problem

- 3rd Normal Form (3NF) modifies the BCNF condition so we do not have to decompose in this problem situation.
- Define: An attribute is *prime* if it is a member of any key.
- *X ->A* violates 3NF if and only if *X* is not a superkey, and also *A* is not prime.
- I.e. a relation R is in third normal form just if for every nontrivial FD X->A, either X is a superkey *or* A is prime (is a member of some key).
  - So this is weaker than BCNF.
  - I.e. if a relation is in BCNF then it must be in 3NF, but not necessarily vice versa

# Example: 3NF

- In our problem situation with FD's *AB ->C* and *C ->B*, we have keys *AB* and *AC*.
- Thus *A*, *B*, and *C* are each prime.
- Although *C ->B* violates BCNF, it does not violate 3NF.

# What 3NF and BCNF Give You

- There are two important properties of a decomposition:
  1. *Lossless Join*: If relation $R$ is decomposed into $R_1$, $R_2$,…, $R_k$, it should be possible to reconstruct $R$ exactly from $R_1$, $R_2$,…, $R_k$.
     - I.e. in decomposing, no information in the original relation is lost
  2. *Dependency Preservation*: It should be possible to check in the projected relations whether all the original FD's are satisfied.
     - I.e. in decomposing, no information given in the functional dependencies is lost.

# 3NF and BCNF -- Continued

- We can get (1) with a BCNF decomposition.
- We can get both (1) and (2) with a 3NF decomposition.
- But we can't always get (1) and (2) with a BCNF decomposition.
  - street/city/p.c. is an example.

# Testing for a Lossless Join

- Ask: If we project $R$ onto $R_1$, $R_2$,…, $R_k$, can we recover $R$ by rejoining?
- Clearly, any tuple in $R$ can be recovered from its projected fragments.
- So the only question is:

  When we rejoin, do we ever get back something we didn't have originally?

- Below, projecting onto (AB) and (BC), and joining introduces tuples (1,2,1) and (3,2,3).

  Such a project/join is called *lossy*.

| A | B | C |
|---|---|---|
| 1 | 2 | 3 |
| 3 | 2 | 1 |

# The Chase Test

- Assume $R$ is decomposed into $R_1$, $R_2$,…, $R_k$
- Suppose tuple $t$ is in the join.
- Then $t$ is the join of projections of some tuples of $R$, one for each $R_i$ of the decomposition.
- Can we use the given FD's to show that one of the tuples in the original relation must be $t$ ?

# The Chase – (2)

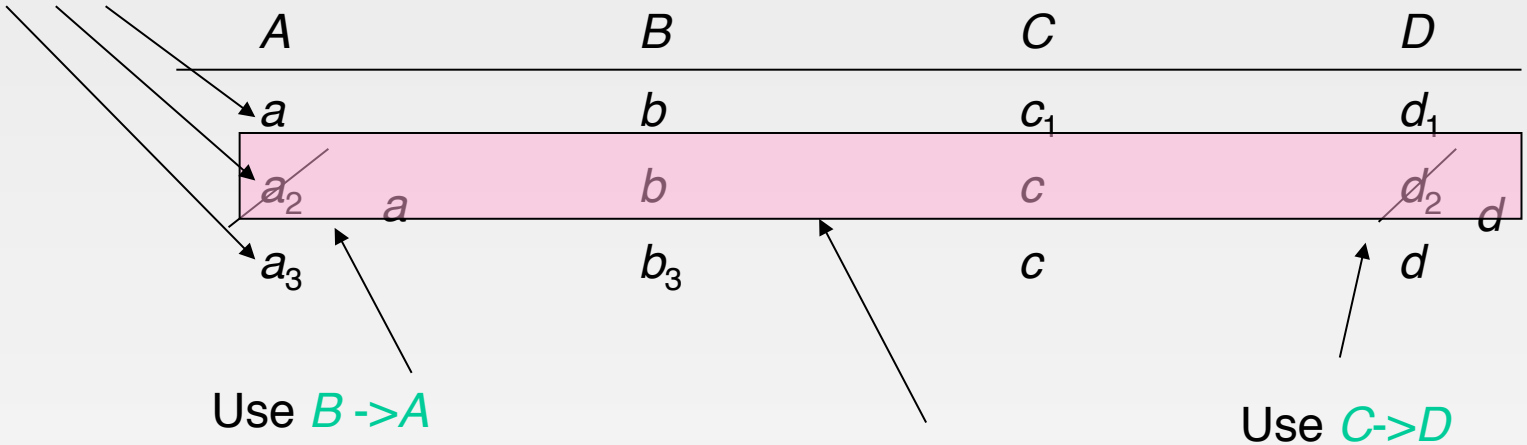
- Start by assuming $t = abc\ldots$ is in the join of the projected relations.
- For each $i$, there is a tuple $s_i$ of $R$ that has $a$, $b$, $c$,… in the attributes of $R_i$.
  - $s_i$ can have any values in other attributes.
- We'll use the same letters as in $t$, but with a subscript, for these components.

# Example: The Chase

- Let *R* = *ABCD*, and the decomposition be *AB*, *BC*, and *CD*.
- Let the given FD's be *C->D* and *B ->A*.
- Suppose the tuple t = abcd is the join of tuples projected onto *AB*, *BC*, *CD*.
- Then there are tuples $abc_1d_1$, $a_2bcd_2$, $abc_3d_3$ in R that are projected and joined to give abcd.

# The *Tableau*

The tuples of R pro-
jected onto AB, BC, CD.



# Summary of the Chase

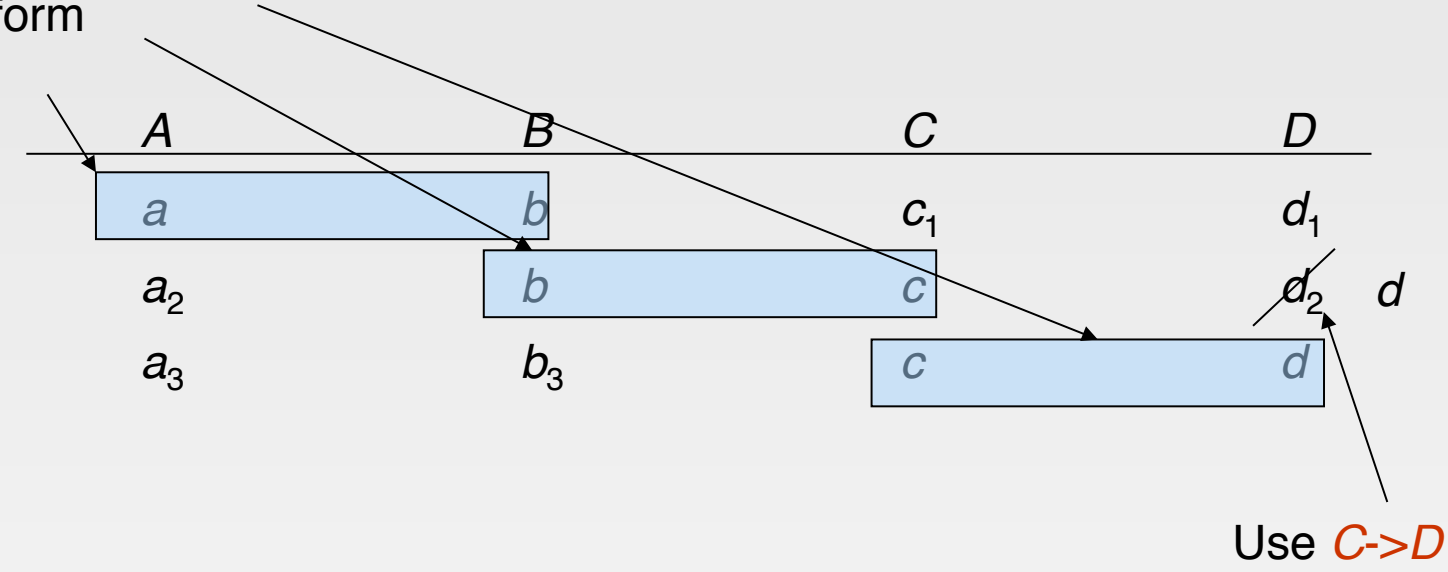
1. If two rows agree in the left side of a FD, make their right sides agree too.
2. Always replace a subscripted symbol by the corresponding unsubscripted one, if possible.
3. If we ever get an unsubscripted row, we know any tuple in the project-join is in the original (the join is lossless).
4. Otherwise, the final tableau is a counterexample.

# Example: Lossy Join

- Same relation $R = ABCD$ and same decomposition.
- But with only the FD $C \rightarrow D$.

# The *Tableau*

These projections rejoin to form *abcd*.




These three tuples are an example that shows that the join is lossy. *The tuple abcd* is not in *R*, but we can project and rejoin to get *abcd*.

# 3NF Synthesis Algorithm

- We can always construct a decomposition into 3NF relations with a lossless join and dependency preservation.
- Need *minimal basis* for the FD's.
- A *basis* for a set of FD's S is a set of FD's that is equivalent to S.
- A *minimal basis* is a basis that satisfies the constraints:
  1. Right sides are single attributes.
  2. No attribute can be removed from a left side.
  3. No FD can be removed
- A minimal basis is also called a *canonical cover*.

# Constructing a Minimal Basis

1. Split right sides of FDs.
2. Repeatedly try to remove an attribute from a left side and see if the resulting FD's are equivalent to the original.
   - I.e. A is redundant in AX->B wrt F if F implies X->B
3. Repeatedly try to remove an FD and see if the remaining FD's are equivalent to the original.
   - I.e. X->A is redundant wrt F if F - {X->A} implies X->A

- Aside: The steps need to be done in the above order.

# 3NF Synthesis – (2)

- Determine a minimal basis.
- Form one relation for each FD in the minimal basis.
  - Schema is the union of the left and right sides of each FD.
  - An optimisation: If there are >1 FDs in the minimal basis with the same LHS, they can be combined.
    - I.e. for X -> A, X -> B, can produce the relation XAB
- If none of the relations above is a superkey for the original relation R, then add one relation whose schema is a key for R.

# Example: 3NF Synthesis

- Relation R = ABCD.
- FD's *A->B* and *A->C*.
- These FDs form a minimal basis
- Decomposition: AB and AC from the FD's, plus AD for a key.
  - As noted, we can also first combine FDs with the same LHS
  - I.e. it's better is to decompose to ABC and AD.

# Another Example

- Relation R = ABC
- FD's AB->C and C->B. (These form a minimal basis.)
- Strictly speaking we get ABC and BC.
- But it *never* makes sense to have a relation whose schema is a subset of another, so we drop BC.

# Why the 3NF Synthesis Algorithm Works

- Preserves dependencies: Each FD from a minimal basis is contained in a relation, thus preserved.
- Lossless Join: Use the chase to show that the row for the relation that contains a key can be made all-unsubscripted variables.
- 3NF: Hard part – a property of minimal bases.

# Other Normal Forms

- First Normal Form says that attribute values are atomic
- Second Normal Form is a restricted version of 3NF and is of historical interest only
- Fourth Normal Form deals with *multivalued dependencies* (MVD's).

# Definition of MVD

- A *multivalued dependency* (MVD) on *R* is an assertion that two attributes or sets of attributes, are independent of each other.
- The MVD *X* ->->*Y* says that if two tuples of *R* agree on all the attributes of *X*, then their components in *Y* may be swapped, and the result will be two tuples that are also in the relation.
- I.e., for each value of *X*, the values of *Y* are independent of the values of *R*-(*X*-*Y*).

# Example: MVD

Customers(name, addr, phones, beersLiked)

- A customer's phones are independent of the beers they like.
  - name->->phones and name ->->beersLiked.
- Thus, each of a customer's phones appears with each of the beers they like in all combinations.
- This repetition is unlike FD redundancy.
  - name->addr is the only FD.
- In fact, note that Customers(name, phones, beersLiked) is in BCNF, though there potential redundancy due to the MVD.

# Tuples Implied by name->->phones

If we have tuples:

| name | addr | phones | beersLiked |
|---|---|---|---|
| sue | a | p1 | b1 |
| sue | a | p2 | b2 |
| sue | a | p2 | b1 |
| sue | a | p1 | b2 |

Then these tuples must also be in the relation.

# Fourth Normal Form

- The redundancy that comes from MVD's is not removable by putting the database schema in BCNF.
- There is a stronger normal form, called 4NF, that (intuitively) treats MVD's as FD's when it comes to decomposition, but not when determining keys of the relation.

# Decomposition and 4NF

- Decompositoin into 4NF is much like BCNF decomposition
- A relation *R* is in *4NF* if:

  If *X* ->-> *Y* is a nontrivial MVD, then *X* is a superkey.
- *X* ->-> *Y* is a 4NF violation if
  - X ->-> Y is nontrivial but
  - *X* is not a superkey.
- If *X* ->-> *Y* is a 4NF violation for relation *R*, we can decompose *R* using the same technique as for BCNF.
  1. *XY* is one of the decomposed relations.
  2. $R - (X^+ - X)$ is the other.

# Decomposition: Summary

- We have focussed on BCNF and 3NF
- Goals of a decomposition:
  1. Elimination of anomalies
  2. Recoverability of information (i.e. lossless join)
  3. Preservation of dependencies
- BCNF gives us 1 and 2 but not necessarily 3
- 3NF gives us 2 and 3 but not necessarily 1
- There is no way to guarantee all three at the same time.

# End: Design Theory for Relational Databases