Relational Algebra

Basic Operations

Algebra of Bags

What is an "Algebra"

- Mathematical system consisting of:
  - *Operands*
    - A domain of *objects*
  - *Operators*
    - Symbols denoting *procedures* (or operations) that construct values from given values.
  - Example: Boolean algebra.
    - Others?

What is Relational Algebra?

- An algebra whose operands are *relations* or *variables* that represent *relations*.
- Operators are designed to do the most common things that we need to do with relations in a database.
  - The result is an algebra that can be thought of as an abstract *query language* for relations.
- I.e. with the relational algebra, you have relations and operators to produce other relations
- Consider: What is a good set of operators for relations?

Core Relational Algebra

- Union, intersection, and difference.
  - Usual set operations, but *both operands must have the same relation schema*.
- Selection: picking certain rows.
- Projection: picking certain columns.
- Products and joins: compositions of relations.
- Renaming of relations and attributes.

Example: R3 := R1 – R2

R1(A, B)

A	B
1	2
3	4

R2(A, B)

A	B
1	1
3	4
9	10

R3(A, B)

A	B
1	2

Selection

- R1 := σ_C (R2)
  - *C* is a condition (as in "if" statements) that refers to attributes of R2.
  - R1 is all those tuples of R2 that satisfy *C*.

Example: Selection

Relation Sells:

bar	beer	price
Joe's	Cdn.	2.50
Joe's	Export	2.75
Sue's	Cdn.	2.50
Sue's	Export	3.00

JoeMenu := $\sigma_{bar=\text{“Joe's”}}$(Sells):

bar	beer	price
Joe's	Cdn.	2.50
Joe's	Export	2.75

Projection

- $R1 := \pi_L (R2)$
  - *L* is a list of attributes from the schema of R2.
  - R1 is constructed by looking at each tuple of R2, extracting the attributes on list *L*, in the order specified, and creating from those components a tuple for R1.
  - Eliminate duplicate tuples, if any.
    - This is because a relation is made up of a *set* of tuples

Example: Projection

Relation Sells:

bar	beer	price
Joe's	Cdn.	2.50
Joe's	Export	2.75
Sue's	Cdn.	2.50
Sue's	Export	3.00

Prices := $\pi_{beer,price}$(Sells):

beer	price
Cdn.	2.50
Export	2.75
Export	3.00

Extended Projection

- Using the same π_L operator, we allow the list L to contain arbitrary expressions involving attributes:
  1. Arithmetic on attributes, e.g., *A+B->C*.
     - So A -> B can be used to rename attribute A.
  2. Can also duplicate occurrences of the same attribute.

Example: Extended Projection

R = (

A	B
1	2
3	4

)

$\pi_{A+B\text{->}C,A,A}$ (R) =

C	A1	A2
3	1	1
7	3	3

Cartesian Product

- Also called *cross product* or simply *product*.
- R3 := R1 X R2
  - Pair each tuple t1 of R1 with each tuple t2 of R2.
  - The concatenation t1t2 is a tuple of R3.
  - Schema of R3 is the attributes of R1 and then R2, in order.
  - But beware: attribute *A* of the same name in R1 and R2 -- use R1.*A* and R2.*A*.

Example: R3 := R1 X R2

R1(A, B)

A	B
1	2
3	4

R2(B, C)

B	C
5	6
7	8
9	10

R3(A, R1.B, R2.B, C)

A	R1.B	R2.B	C
1	2	5	6
1	2	7	8
1	2	9	10
3	4	5	6
3	4	7	8
3	4	9	10

Theta-Join

- R3 := R1 $\bowtie_C$ R2
  - Take the product R1 X R2.
  - Then apply σ_C to the result.
  - Aside: This means that theta-join is a *redundant* operator
- As for σ, C can be any boolean-valued condition.
  - Historic versions of this operator allowed only A θ B, where θ is =, <, etc.; hence the name "theta-join."

Example: Theta Join

Sells(

bar,	beer,	price
Joe's	Cdn.	2.50
Joe's	Ex.	2.75
Sue's	Cdn.	2.50
Sue's	G.I.	3.00

)

Bars(

name,	addr
Joe's	Maple St.
Sue's	River Rd.

)

BarInfo := Sells $\bowtie_{Sells.bar = Bars.name}$ Bars

BarInfo(

bar,	beer,	price,	name,	addr
Joe's	Cdn.	2.50	Joe's	Maple St.
Joe's	Export	2.75	Joe's	Maple St.
Sue's	Cdn.	2.50	Sue's	River Rd.
Sue's	G.I.	3.00	Sue's	River Rd.

)

Natural Join

- A useful join variant (*natural* join) connects two relations by:
  - Equating attributes of the same name, and
  - Projecting out one copy of each pair of equated attributes.
- Denoted R3 := R1 $\bowtie$ R2.

Example: Natural Join

Sells(

bar,	beer,	price
Joe's	Cdn.	2.50
Joe's	Export	2.75
Sue's	Cdn.	2.50
Sue's	G.I.	3.00

)

Bars(

bar,	addr
Joe's	Maple St.
Sue's	River Rd.

)

BarInfo := Sells $\bowtie$ Bars

Note: Bars.name has become Bars.bar to make the natural join "work."

BarInfo(

bar,	beer,	price,	addr
Joe's	Cdn.	2.50	Maple St.
Joe's	Export	2.75	Maple St.
Sue's	Cdn.	2.50	River Rd.
Sue's	G.I.	3.00	River Rd.

)

Renaming

- The ρ operator gives a new schema to a relation.
- R1 := $\rho_{R1(A1,\ldots,An)}$(R2) makes R1 be a relation with attributes A1, …,A*n* and the same tuples as R2.
- Simplified notation: R1(A1,…,A*n*) := R2.

Example: Renaming

Bars(

name,	addr
Joe's	Maple St.
Sue's	River Rd.

)

$\rho_{R(bar,addr)}$(Bars) or R(bar, addr) := Bars

R(

bar,	addr
Joe's	Maple St.
Sue's	River Rd.

)

Relationships Among Operations

- Some operations that we've seen can be expressed in terms of other relational algebra operations.
- E.g. We've seen that theta-join can be expressed by product and selection
- Another example: $R \cap S = R - (R - S)$
- $R \bowtie S = \pi_L (\sigma_C (R \times S))$

  where

  - C is a conjunction of elements of the form R.A = S.A for all attributes common to R and S, and
  - L is a list of the attributes in the schema of R followed by those attributes of S not in R.

Building Complex Expressions

- Each operation has a relation as its value
- Can combine operations with parentheses and precedence rules.
- Three notations for complex expressions, just as in arithmetic:
  1. Sequences of assignment statements.
  2. Expressions with several operators.
  3. Expression trees.
- We'll use 1 and 2, not 3.

1. Sequences of Assignments

- Create temporary relation names.
- Renaming can be implied by giving relations a list of attributes.
- Example: R3 := R1 $\bowtie_C$ R2 can be written:

  R4 := R1 X R2

  R3 := σ_C(R4)

2. Expressions in a Single Assignment

- Example: the theta-join R3 := R1 $\bowtie_C$ R2 can be written:

  R3 := σ_C (R1 X R2)

- Precedence of relational operators:
  1. σ, π, ρ (highest)
  2. X, $\bowtie$
  3. $\cap$
  4. $\cup$, —
- When in doubt, or for clarity, use parentheses.

3. Expression Trees

- Leaves are operands --- either variables standing for relations or particular, constant relations.
- Non-leaf nodes are operators, applied to their child or children.

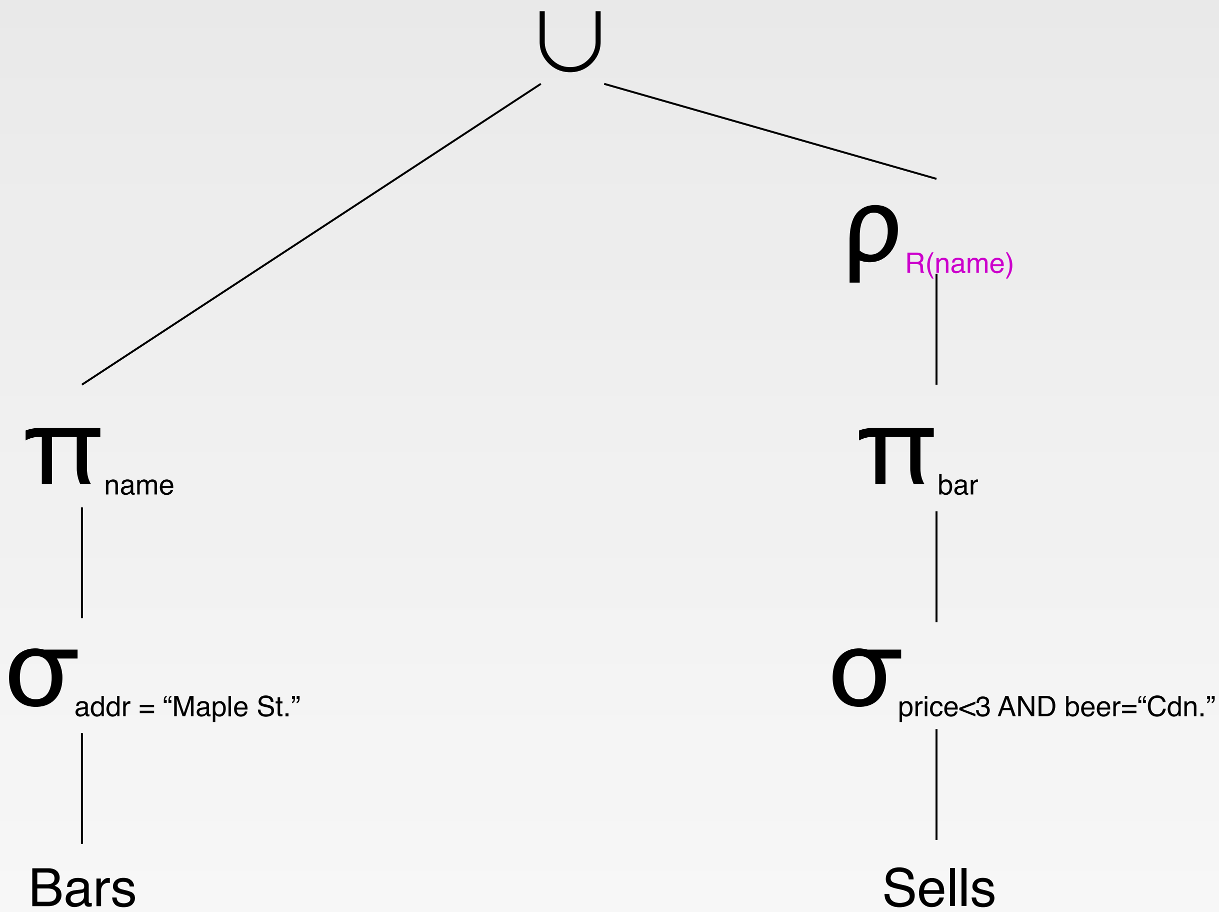
Example: Tree for a Query

- Using the relations

  Bars(name, addr) and Sells(bar, beer, price),

  find the names of all the bars that are either on Maple St. or sell Cdn. for less than $3.

As a Tree:



∪
ρ R(name)
π name
π bar
σ addr = “Maple St.”
σ price<3 AND beer=“Cdn.”
Bars
Sells

Example: Self-Join

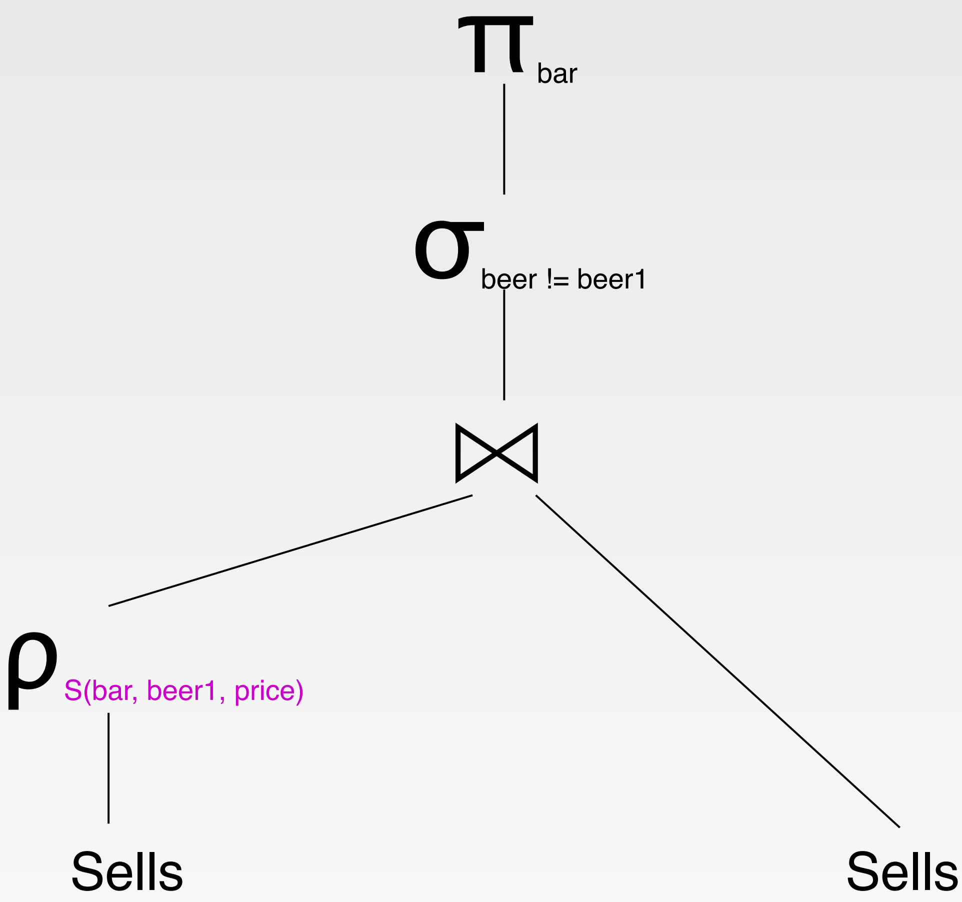
- Using Sells(bar, beer, price), find the bars that sell two different beers at the same price.

Example: Self-Join

- Using Sells(bar, beer, price), find the bars that sell two different beers at the same price.
- Strategy:
  1. by renaming, define a copy of Sells, called

     S(bar, beer1, price).
  2. The natural join of Sells and S consists of quadruples (bar, beer, beer1, price) such that the bar sells both beers at this price.

The Tree



π_{bar}
$\sigma_{beer\ !=\ beer1}$
$\bowtie$
$\rho_{S(bar,\ beer1,\ price)}$
Sells
Sells

Schemas for Results

- Union, intersection, and difference: the schemas of the two operands must be the same, so use that schema for the result.
- Selection: schema of the result is the same as the schema of the operand.
- Projection: the list of attributes tells us the schema.

Schemas for Results --- (2)

- Product: schema is the attributes of both relations.
  - Use R.*A*, etc., to distinguish two attributes named *A*.
- Theta-join: same as product.
- Natural join: union of the attributes of the two relations.
- Renaming: the operator tells the schema.

Relational Algebra on Bags

- A *bag* (or *multiset*) is like a set, but an element may appear more than once.
- Example: {1,2,1,3} is a bag.
- Example: {1,2,3} is also a bag that happens to be a set.

Why Bags?

- SQL, the most important query language for relational databases, is actually a bag language.
- Some operations, like projection, are more efficient on bags than sets.

Operations on Bags

- Selection applies to each tuple, so its effect on bags is like its effect on sets.
- Projection also applies to each tuple, but as a bag operator, we do not eliminate duplicates.
- Products and joins are done on each pair of tuples, so duplicates in bags have no effect on how we operate.

Example: Bag Selection

R(A, B)

A	B
1	2
5	6
1	2

$\sigma_{A+B<5}(R) =$

A	B
1	2
1	2

Example: Bag Projection

R(

A,	B
1	2
5	6
1	2

)

$\pi_A (R) =$

A
1
5
1

Example: Bag Product

R(

A,	B
1	2
5	6
1	2

)

S(

B,	C
3	4
7	8

)

R X S =

A	R.B	S.B	C
1	2	3	4
1	2	7	8
5	6	3	4
5	6	7	8
1	2	3	4
1	2	7	8

Example: Bag Theta-Join

R(

A,	B
1	2
5	6
1	2

)

S(

B,	C
3	4
7	8

)

$R \bowtie_{R.B<S.B} S =$

A	R.B	S.B	C
1	2	3	4
1	2	7	8
5	6	7	8
1	2	3	4
1	2	7	8

Bag Union

- An element appears in the union of two bags the sum of the number of times it appears in each bag.
- Example: $\{1,2,1\} \cup \{1,1,2,3,1\} = \{1,1,1,1,1,2,2,3\}$

Bag Intersection

- An element appears in the intersection of two bags the minimum of the number of times it appears in either.
- So calculate bag intersection by "pairing off" elements from each operand
- Example: $\{1,2,1,1\} \cap \{1,2,1,3\} = \{1,1,2\}$.

Bag Difference

- An element appears in the difference $A - B$ of bags as many times as it appears in *A*, minus the number of times it appears in *B*.
  - But never less than 0 times.
- Example: $\{1,2,1,1\} - \{1,2,3\} = \{1,1\}$.

Beware: Bag Laws != Set Laws

- Some, but *not all* algebraic laws that hold for sets also hold for bags.
- Example: the commutative law for union ($R \cup S = S \cup R$) holds for bags.
  - Since addition is commutative, adding the number of times x appears in R and S doesn't depend on the order of R and S.

Example: A Law That Fails

- Set union is *idempotent*, meaning that $S \cup S = S$.
- However, for bags, if x appears n times in S, then it appears $2n$ times in $S \cup S$.
- Thus $S \cup S$!= S in general.
  - e.g., {1} ∪ {1} = {1,1} != {1}.

End: Relational Algebra