CMPT 307-08-2 Assignment 8

(From lecture on June 24, 2008)

Deadline: July 8, 5:30pm

Problem 8.1. Prove that the class of all functions from U to $\{0, 1, \ldots, m-1\}$ is universal.

Problem 8.2. Write a pseudo-code for the modified **Hash-Insert** able to handle also the special value DELETED, and for the procedure **Hash-Delete**.

Problem 8.3. Consider a quadratic probing scheme with constants $m = 2^t$ for some integer t (i.e., m is a power of 2) and $c_1 = c_2 = 1/2$. Prove that the function

$$h(k, i) = (h'(k) + c_1 i + c_2 i^2) \bmod m$$

is indeed a hash function, i.e., prove that all probe sequences are permutations of $\langle 0, 1, \ldots, m-1 \rangle$. *Hint:* Congruence $a/2 \equiv b \mod 2^t$ makes sense only if a is even, and is equivalent to congruence $a \equiv 2b \mod 2^{t+1}$.

Problem 8.4. Consider a random variable X having only positive integer values (i.e., it's mapping the probability space to the set of natural numbers). Prove that

$$E[X] = P(X \geq 1) + P(X \geq 2) + \cdots = \sum_{i=1}^{\infty} P(X \geq i)$$

Hint: events $\{X = 1\}$, $\{X = 2\}$, $\{X = 3\}$, ..., are mutually exclusive (disjoint), hence

$$P(X \geq i) = \sum_{j=i}^{\infty} P(X = j)$$