CMPT 307-08-2 Assignment 6

(From lecture on June 10, 2008)

Deadline: June 17, 5:30pm

Problem 6.1. Suppose that instead of swapping element $A[i]$ with a random element from the subarray $A[i \dots n]$, we swapped it with a random element from anywhere in the array:

```
Permute-With-All($A[1 \dots n]$)
for $i \leftarrow 1$ to $n$ do
    swap $A[i] \leftrightarrow A[\textbf{Random}(1, n)]$
end for
return $A$
```

Show that this code doesn't produce a uniform random permutation for all sizes of inputs. That is, find an integer n, for which the above algorithm doesn't produce a uniform random permutation. Justify your answer.

Problem 6.2. Roll 3 fair 6-sided dice. Consider random variable X equal to the sum of outcomes on all dice. (Hence, X has values from 3 to 18.) Calculate $E[X]^2$ and $E[X^2]$.

Problem 6.3. Suppose that the **for** loop 13–16 is rewritten so that the loops start with $j = 1$ and end with $j = n$, i.e., the line 13 is changed to

```
for $j \leftarrow 1$ to $n$ do
```

Show that the algorithm still works properly. Is the modified algorithm stable?

Problem 6.4. Which of the following sorting algorithms are stable: selection sort, merge sort, heapsort, and quicksort? If the algorithm is not stable, give an example of an input showing it. If the algorithm is stable, prove it.

Problem 6.5. Show how to sort n integers in the range 0 to $n^2 - 1$ in $\mathcal{O}(n)$ time.