CMPT 307-08-2 Assignment 3

(From lecture on May 20, 2008)

Deadline: May 27, 5:30pm

Problem 3.1. Prove by mathematical induction on h that there are at most $\lceil n/2^{h+1} \rceil$ nodes of height h in any n-element heap.
Hint: For the induction step, your induction hypothesis is

- For any n, n-element heap has at most $\lceil n/2^{h} \rceil$ nodes of height $h-1$.

and you want to prove that

- For any n, n-element heap has at most $\lceil n/2^{h+1} \rceil$ nodes of height h.

Hence, if you are proving the claim for a heap H with n elements, then you can apply the induction hypothesis on any heap (for instance on a heap which contains only a part of H).

Problem 3.2. Argue the correctness of **Heap-Increase-Key** using the following loop invariant

- At the start of each iteration of the **while** loop of lines 5–8, the array $A[1 \ldots \text{heap-size}(A)]$ satisfies the max-heap property with possible one exception: $A[i]$ may be larger than $A[\text{Parent}(i)]$.

Problem 3.3. Draw an "efficient" decision tree for inputs with 4 elements.
Hint: The height of the tree should be 5.