CMPT 307-08-2 Assignment 11

(From lecture on July 22, 2008)

Deadline: July 29, 5:30pm

Problem 11.1. Show that the number of full parenthesizations of a product of n matrices, $P(n)$ is in $\Omega(2^n)$.

Problem 11.2. Consider a variant of the matrix-chain multiplication problem in which the goal is to parenthesize the sequence of matrices so as to maximize, rather than minimize, the number of scalar multiplications. Perform all 4 steps to design a Dynamic Programming algorithm.

Problem 11.3. Give an $\mathcal{O}(n+m)$ time algorithm deciding whether a sequence $X = x_1 \ldots x_n$ is a subsequence of $Y = y_1 \ldots y_m$. Remember to explain how you algorithm works!
Note: A DP algorithm for this problem would work in time $O(n.m)$. You will only get a half of the points for such a solution.

Problem 11.4. Give an $\mathcal{O}(n^2)$ time algorithm to find the longest monotonically increasing subsequence of a sequence of n distinct numbers.

Problem 11.5. Show that the above "multiplication" (with min instead of sum and addition instead of multiplication) of matrices is associative, i.e., that for any three matrices A, B and C, we have

$$A \otimes (B \otimes C) = (A \otimes B) \otimes C.$$

Problem 11.6. We are assuming that the graph doesn't contain a negative-weight cycle. What happens if we drop this assumption?

1. Show that if a graph contains a negative-weight cycle then there are two vertices with a shortest path distance $-\infty$.

2. Use matrices $D^{(1)}, \ldots, D^{(n)}$ to identify that the graph contains a negative-weight cycle, as well, to find the length (the number of edges) of a smallest such cycle.