CMPT 307-08-2 Assignment 1

(From lecture on May 6, 2008)

Deadline: May 13, 5:25pm

Problem 1.1. Consider the following sorting algorithm:

BUBBLE-SORT

Input: n numbers in array $A[1], \ldots, A[n]$

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1: for i ← 1 to n do
2:   for j ← n downto i + 1 do
3:     if A[j − 1] > A[j] then
4:       swap A[j − 1] and A[j]
5:     end if
6:   end for
7: end for
```

Find a loop invariant of the main loop and use it to prove that algorithm is correct!

Problem 1.2. Show that for any real constants a and b, where $b > 0$,

(a) $b^{n+a} \in \Theta(b^n)$;

(b) $(n + a)^b \in \Theta(n^b)$.

Problem 1.3. Does $f(n) \in \mathcal{O}(g(n))$ implies

(a) $g(n) \in \mathcal{O}(f(n))$?

(b) $\frac{1}{g(n)} \in \mathcal{O}(\frac{1}{f(n)})$?

If does prove it, if does not, show an example of two functions $f(n)$ and $g(n)$ which satisfy $f(n) \in \mathcal{O}(g(n))$, but do not satisfy the condition (a) (respectively, (b)).

Problem 1.4. Prove that

(a) $\mathcal{O}(g(n)) \cap \Omega(g(n)) = \Theta(g(n))$;

(b) $o(g(n)) \cap \omega(g(n))$ is the empty set.

Problem 1.5. Prove that $n! \in \omega(2^n)$ and $n! \in o(n^n)$.