Queues, Stacks, Graph Traversing

Data Structures and Algorithms
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Graph Reminder

- Vertices and edges
- Nodes and arcs
- Representation of graphs:
  - -- adjacency matrix
  - -- adjacency lists
- Degrees of vertices, indegree, outdegree; regular graphs
- Walks, paths, and cycles; lengths
- Connectivity, connected components
- Trees, root, leaves, parent and child, descendant and ancestor

Graph Traversing

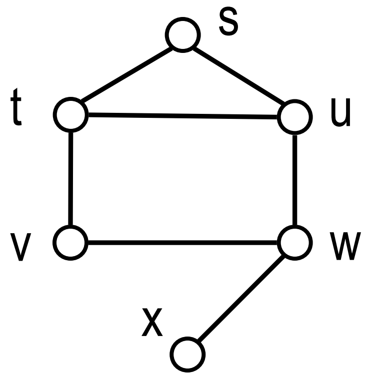
Often we need to visit every vertex of a graph

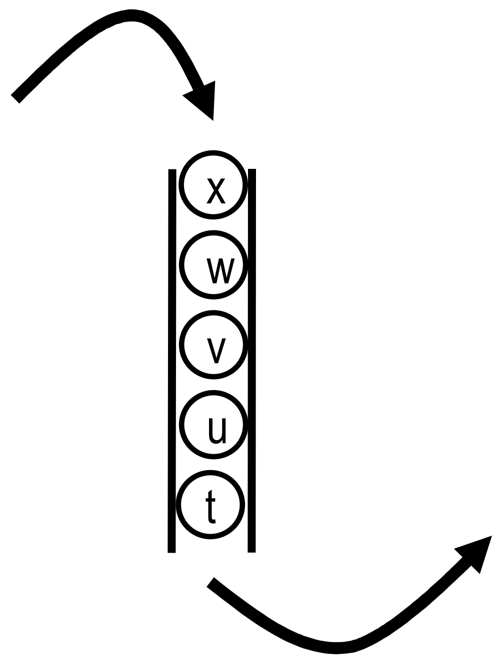
There are many ways to do that, two are most usual: breadth first search and depth first search

In both cases we start with some vertex s

For BFS:

- visit neighbors of s
- then visit neighbors of neighbors in turn
- this data structure is called a queue



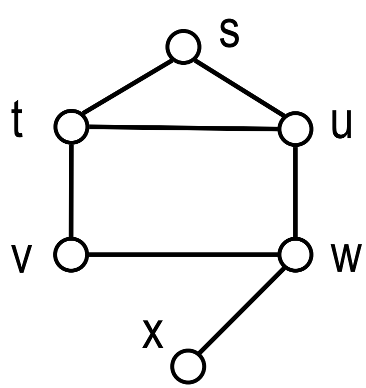


Graph Traversing: BFS

Breadth First Search(G,s)

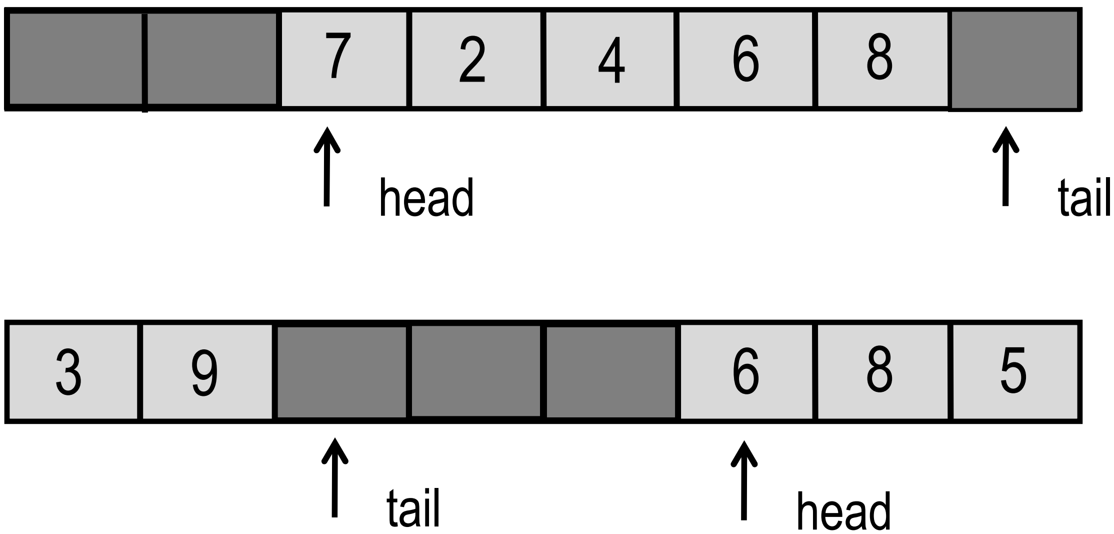
```
set Discov[s]:=true and Discov[v]:=false for v≠s
Enqueue(Q,s)
while Q is not empty do
  set u:=Dequeue(Q)
  for each (u,v)∈E do
     if Discov[v]=false then do
        set Discov[v]:=true;
        Enqueue(Q,v)
     endif
  endfor
endwhile
```

Example



Queues Through Array

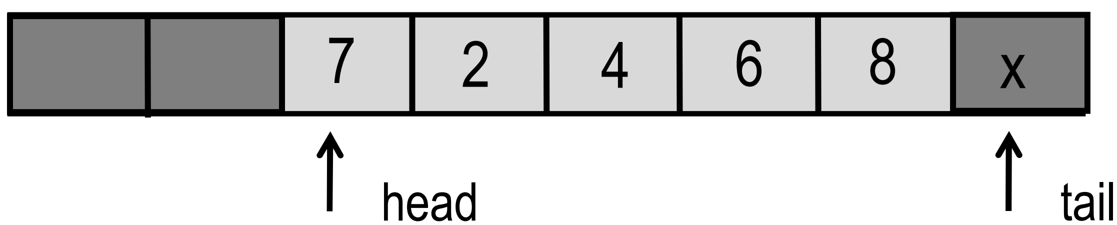
If we store a queue in an array, we need two pointers to the head and to the tail of the queue



Enqueue

Enqueue(G,x)

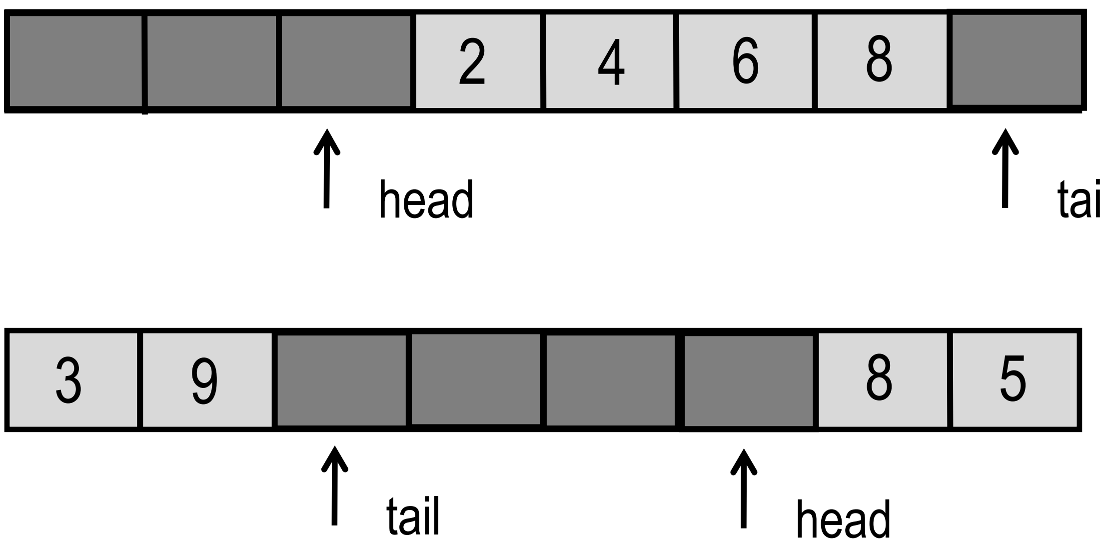
```
set Q[tail[Q]]:=x
if tail[Q]=length[Q] then
   set tail[Q]:=1
else set tail[Q]:=tail[Q]+1
```



Dequeue

Dequeue(G)

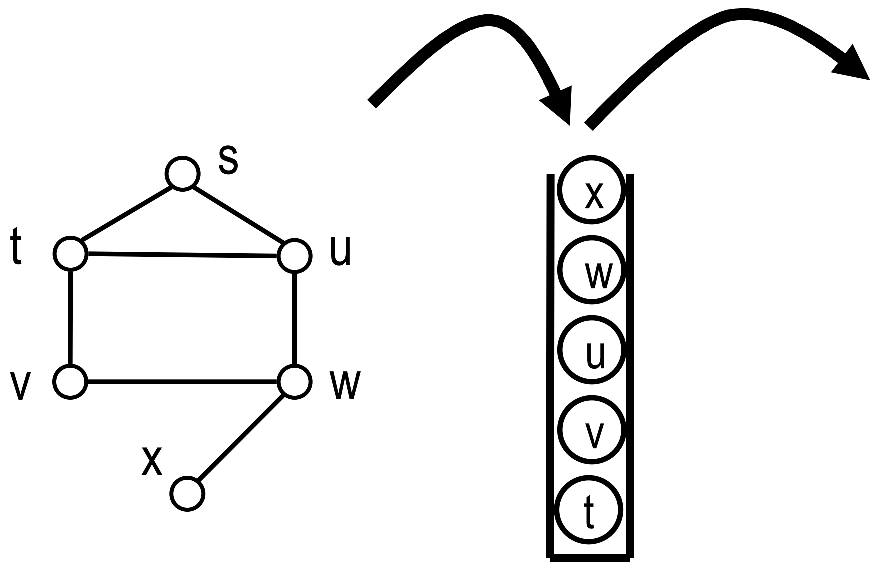
```
set v:=Q[head[Q]]
if head[Q]=length[Q] then
   set head[Q]:=1
else set head[Q]:=head[Q]+1
```



Graph Traversing: DFS

For DFS:

- start with some vertex s
- visit first neighbor of s
- then visit neighbors of that neighbor
- every time consider the neighbors of the last vertex visited
- this data structure is called a stack

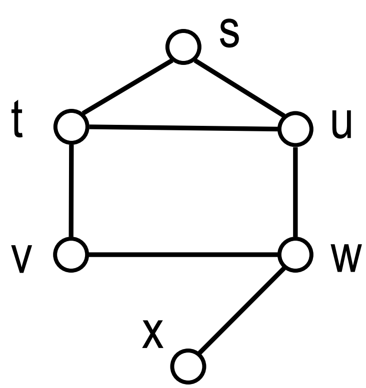


Graph Traversing: DFS

Depth First Search(G,s)

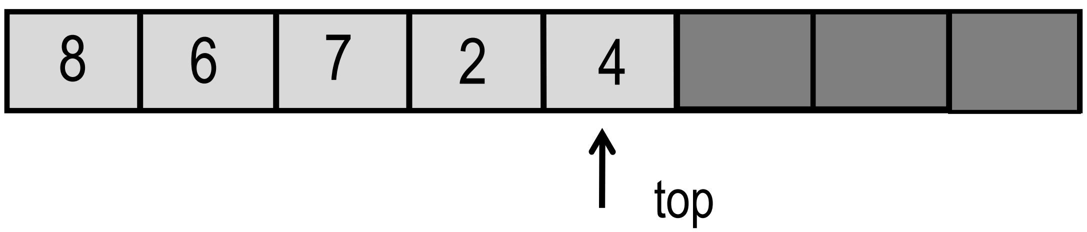
```
set Explor[s]:=true and Explor[v]:=false for v≠s
Push(S,s)
while not Stack-Empty(S) do
  set u:=Pop(S)
  for each (u,v)∈E do
     if Explor[v]=false then do
        set Explor[v]:=true;
        Push(S,v)
     endif
  endfor
endwhile
```

Example



Stacks Through Array

If we store a stack in an array, we need a pointer to the top of the stack



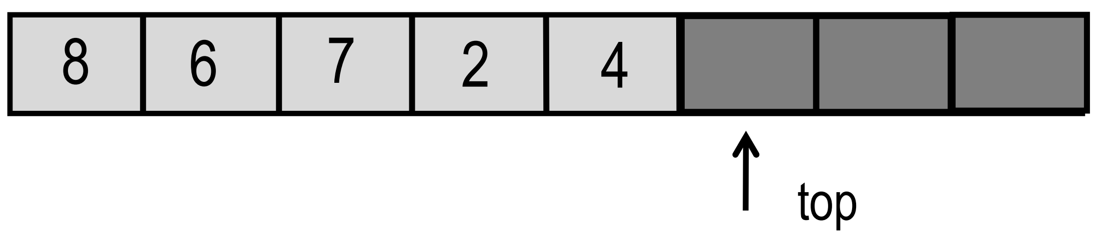
Stack-Empty(S)

```
if top[S]=0 then
   return true
else return false
```

Push

Push(S,x)

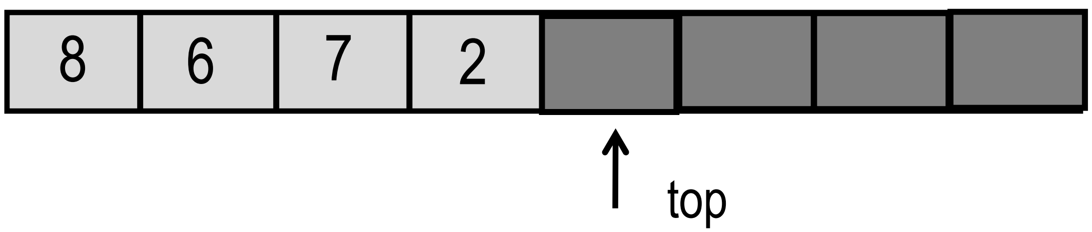
```
set top[S]:=top[S]+1
set S[top[S]]:=x
```



Pop

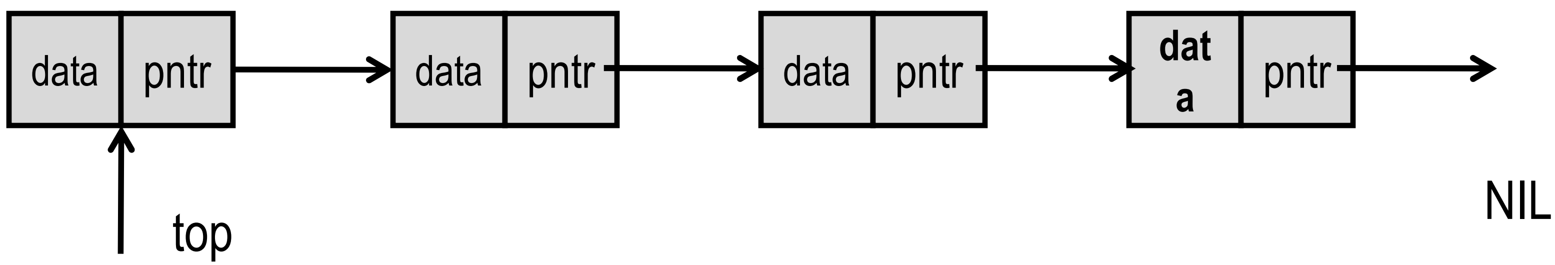
Pop(S)

```
if Stack-Empty(S) then
   error “underflow”
else do
   set top[S]:=top[S]-1
   return S[top[S]+1]
```



Stacks Through Pointers and Objects

Stacks can also be stored using pointers and objects



Stack-Empty(S)

```
if top[S]=Nil then
   return true
else return false
```

Push and Pop

Push(S,x)

```
set next[x]:=top[S]
set top[S]]:=x
```

Pop(S)

```
set t:=top[S]
set top[S]:=next[top[S]]
return t
```

Stacks Through Pointers and Objects

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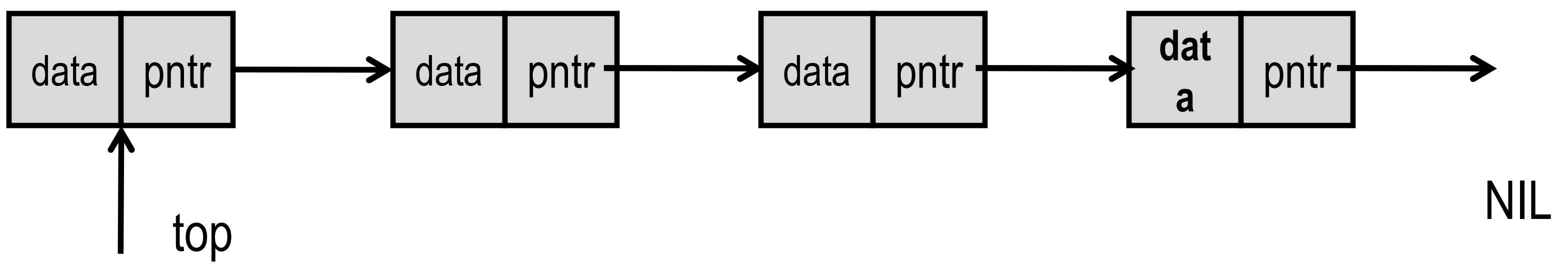


Stack-Empty(S)

```
if top[S]=Nil then
   return true
else return false
```

Push and Pop

Push(S,x)

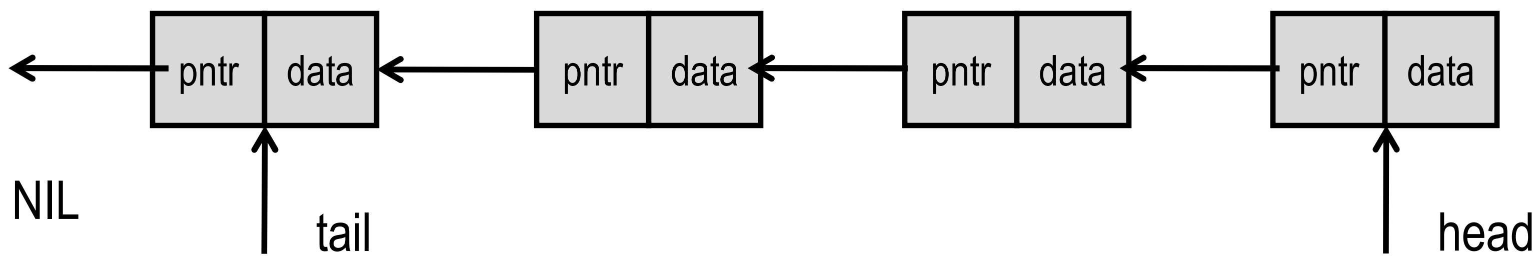
```
set next[x]:=top[S]
set top[S]:=x
```

Pop(S)

```
set t:=top[S]
set top[S]:=next[top[S]]
return t
```

Queues Through Pointers and Objects

Queues can also be stored using pointers and objects



Enqueue and Dequeue

Enqueue(Q,x)

```
set next[tail[Q]]:=x
set top[S]:=x
```

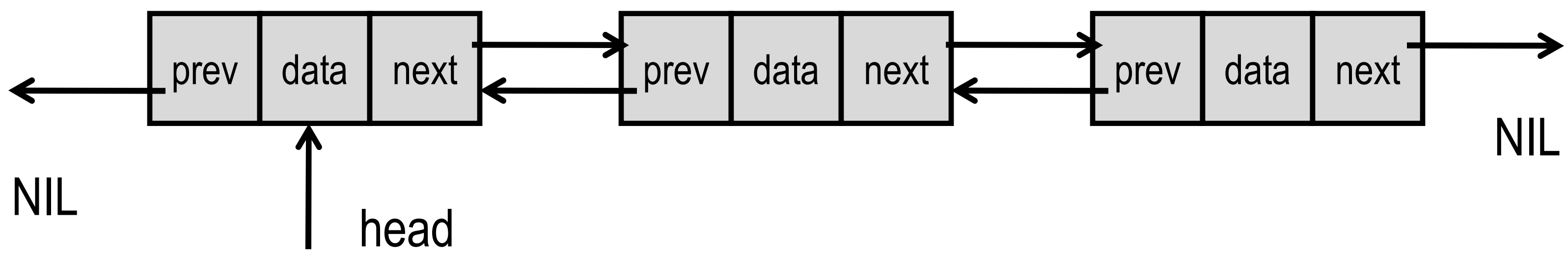
Dequeue(Q)

```
set x:=head[S]
set head[Q]:=next[head[Q]]
return x
```

Doubly Linked Lists

To run, say, insertion sort, just a list (or queue, or stack) is not enough, as we need to move along the list back and forth

A doubly linked list is used



Operations:

List-Search

List-Insert

List-Delete

Search

List-Search(L,k)

```
set x:=head[L]
while x≠NIL and data[x]≠k do
   set x:=next[x]
return x
```

Insert and Delete

List-Insert(L,x)

```
set next[x]:=head[L]
if head[L]≠NIL then do
   set prev[head[L]:=x
set head[L]:=x
set prev[x]:=NIL
```

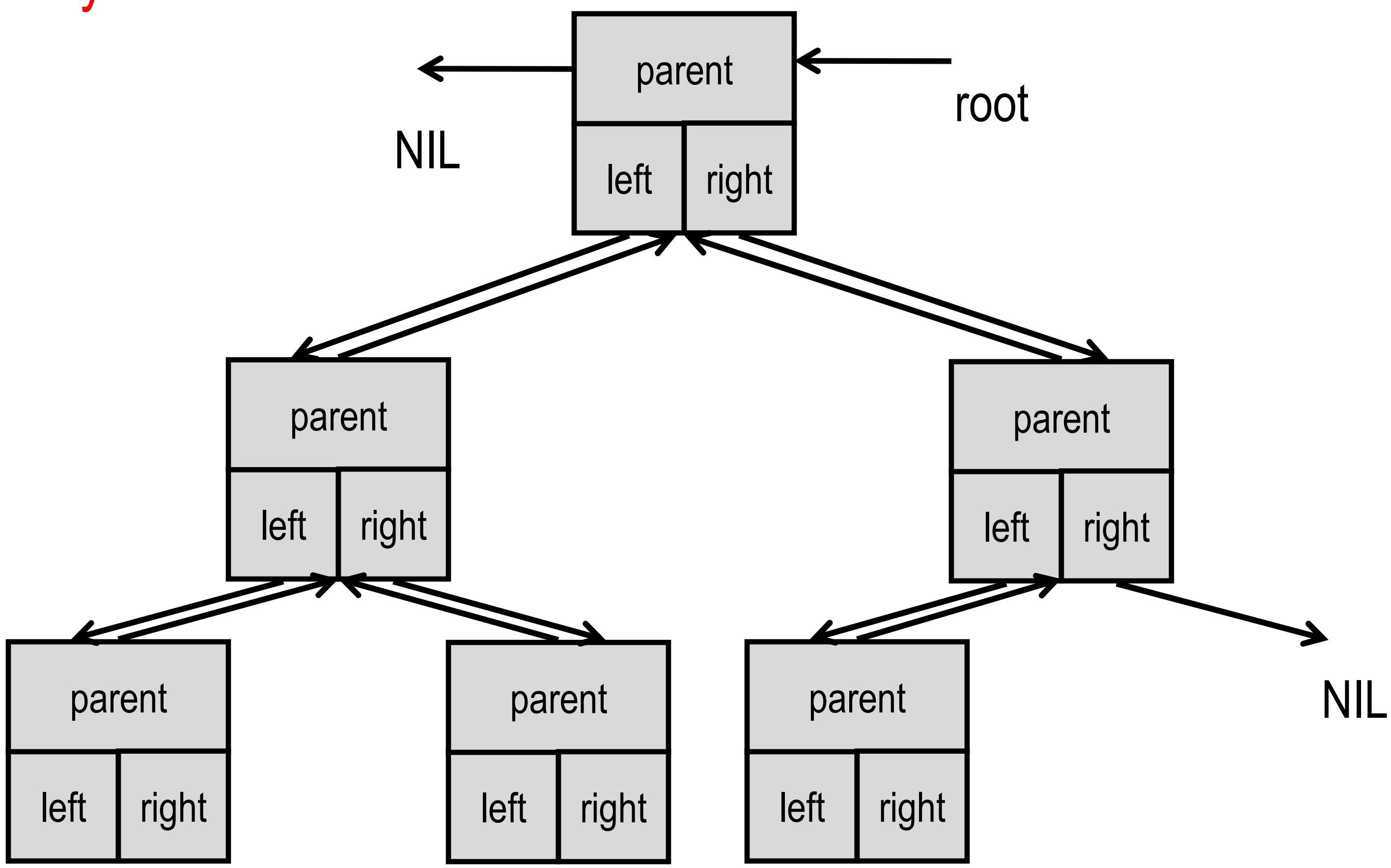
List-Delete(L,x)

```
if prev[x]≠NIL then
   set next[prev[x]]:=next[x]
else
   head[L]:=next[x]
if next[x]≠NIL then
   set prev[next[x]:=prev[x]
```

Binary Rooted Trees

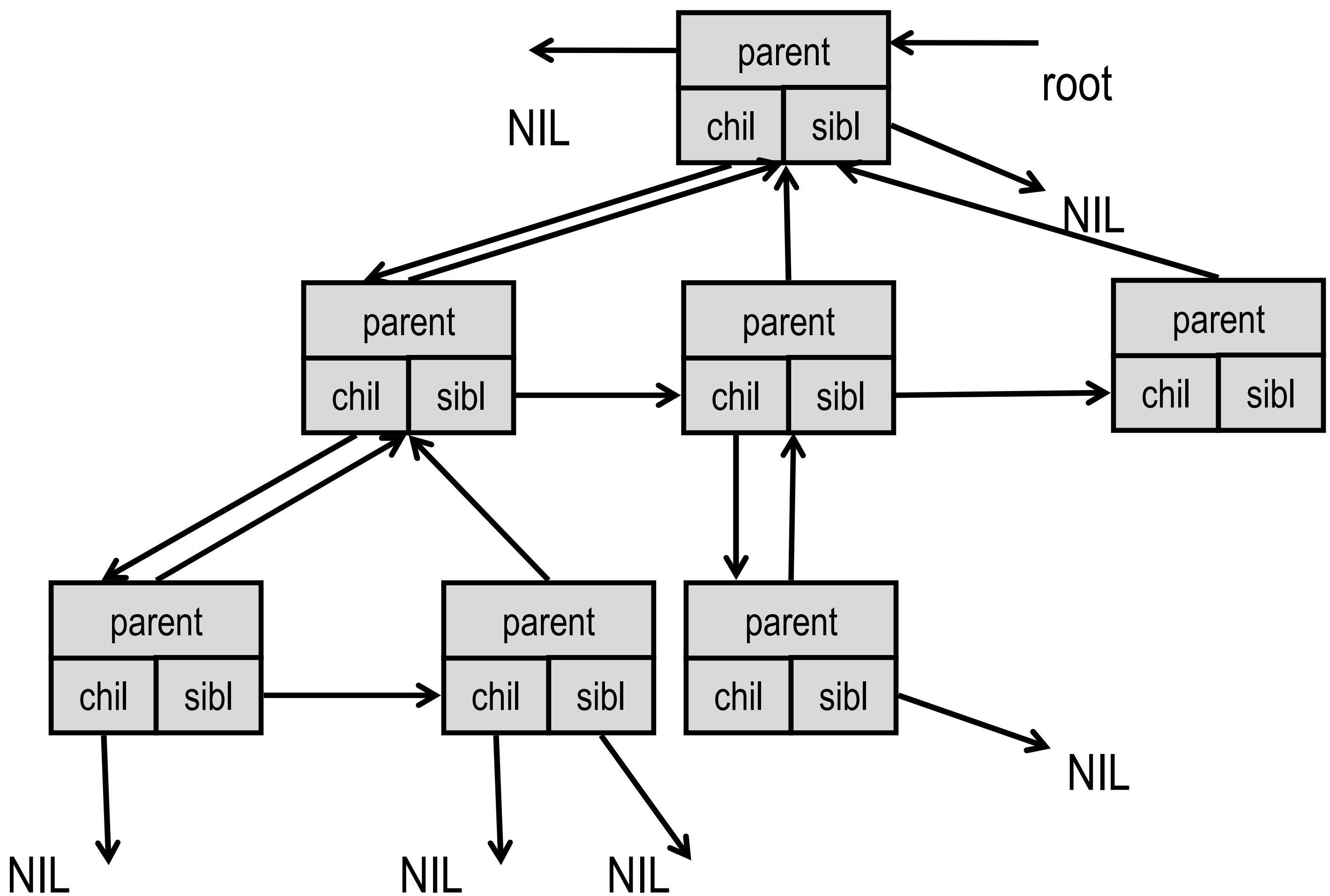
If we need to run heap sort on a sequence of numbers of unpredictable length, we cannot organize heap in an array

A binary tree is used



Arbitrary Rooted Trees

Sometimes a non-binary tree is needed



Homework

Write pseudocode of Insertion Sort if the data is stored in a doubly linked list

Write pseudocode of Heap Sort using the binary tree representation of data rather than arrays