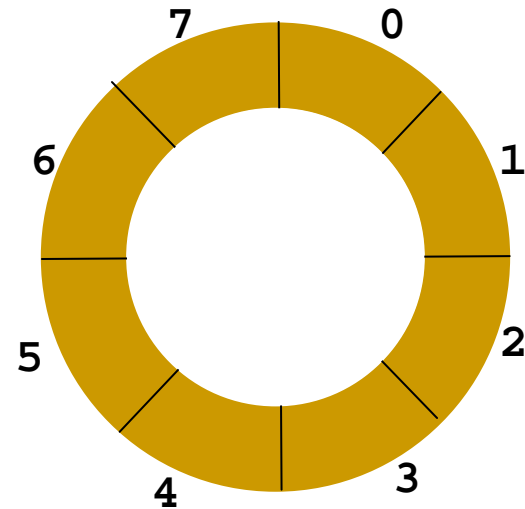




Circular Arrays

- **Neat trick:** use a *circular array* to insert and remove items from a queue in constant time
- The idea of a circular array is that the end of the array “wraps around” to the start of the array





The mod Operator

- The mod operator (%) is used to calculate remainders:
 - $1\%5 = 1$, $2\%5 = 2$, $5\%5 = 0$, $8\%5 = 3$
- mod can be used to calculate the front and back positions in a circular array, therefore avoiding comparisons to the array size
 - The back of the queue is:
 - `(front + count - 1) % items.length`
 - where `count` is the number of items currently in the queue
 - After removing an item the front of the queue is:
 - `(front + 1) % items.length;`



Array Queue Example

front =	0
count =	1

6					
0	1	2	3	4	5

```
//Java Code  
Queue q = new Queue();  
q.enqueue(6);
```

insert item at $(\text{front} + \text{count}) \% \text{items.length}$

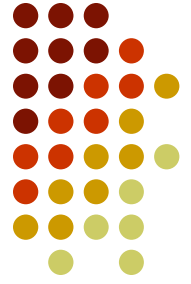


Array Queue Example

front =	0
count =	5

6	4	7	3	8	
0	1	2	3	4	5

```
//Java Code  
Queue q = new Queue();  
q.enqueue(6);  
q.enqueue(4);  
q.enqueue(7);  
q.enqueue(3);  
q.enqueue(8);
```



Array Queue Example

front =	2
count =	4

6	4	7	3	8	9
0	1	2	3	4	5

make front = $(0 + 1) \% 6 = 1$

make front = $(1 + 1) \% 6 = 2$

//Java Code

```
Queue q = new Queue();  
q.enqueue(6);  
q.enqueue(4);  
q.enqueue(7);  
q.enqueue(3);  
q.enqueue(8);  
q.dequeue(); //front = 1  
q.dequeue(); //front = 2  
q.enqueue(9);
```



Array Queue Example

front =	2
count =	5

5		7	3	8	9
0	1	2	3	4	5

**insert at $(\text{front} + \text{count}) \% 6$
 $= (2 + 4) \% 6 = 0$**

//Java Code

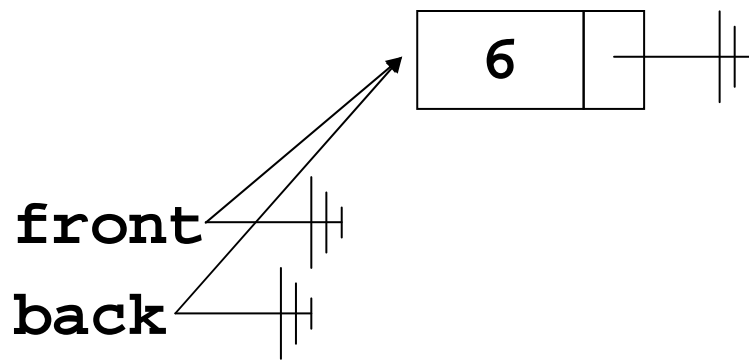
```
Queue q = new Queue();  
q.enqueue(6);  
q.enqueue(4);  
q.enqueue(7);  
q.enqueue(3);  
q.enqueue(8);  
q.dequeue(); //front = 1  
q.dequeue(); //front = 2  
q.enqueue(9);  
q.enqueue(5);
```

Queue: Linked List Implementation



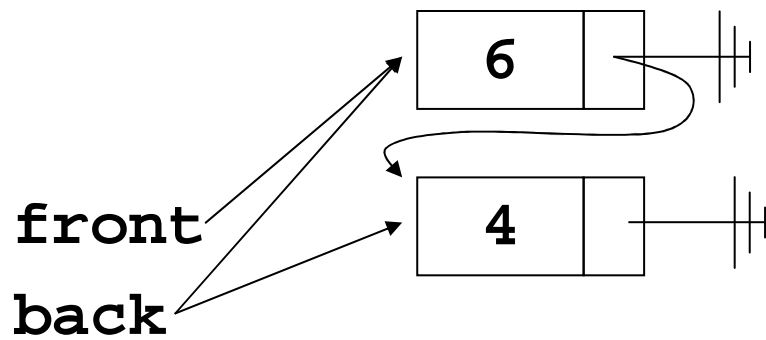
- Removing items from the front of the queue is straightforward
- But we need to insert items at the back of the queue in constant time
 - So cannot traverse through the list
 - By using an additional reference (and a little administration) we can keep track of the node at the back of the queue

List Queue Example



```
//Java Code  
Queue q = new Queue();  
q.enqueue(6);
```

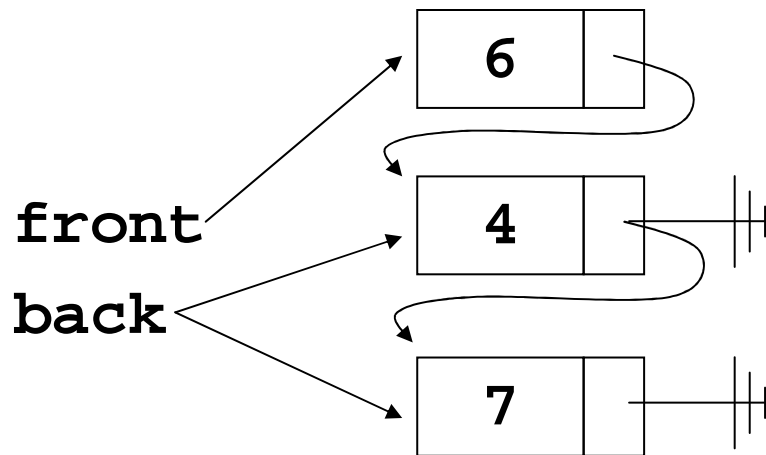

List Queue Example



```
//Java Code
Queue q = new Queue();
q.enqueue(6);
q.enqueue(4);
```



List Queue Example

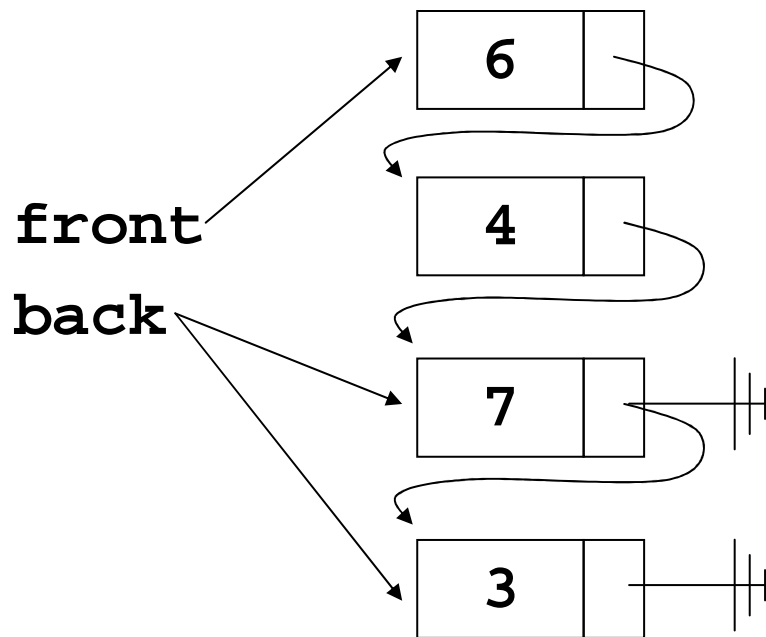


```
//Java Code
```

```
Queue q = new Queue();  
q.enqueue(6);  
q.enqueue(4);  
q.enqueue(7);
```



List Queue Example

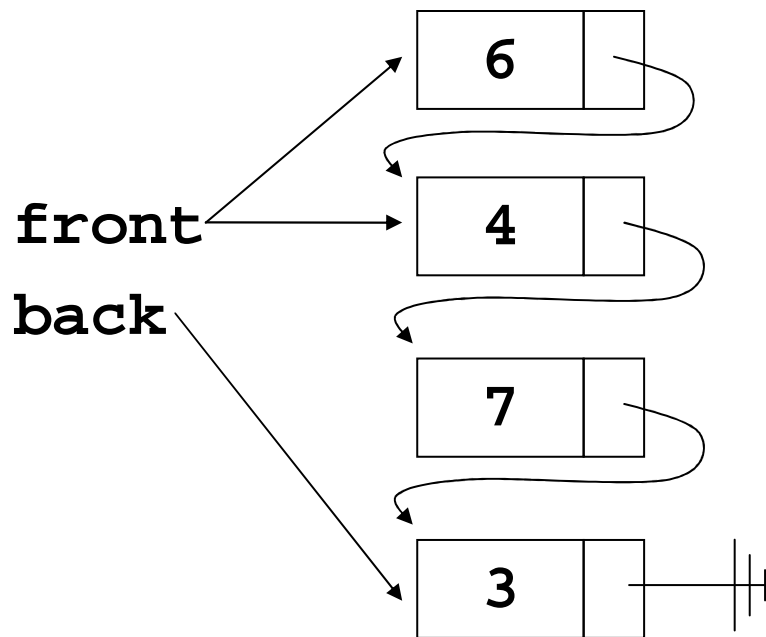


//Java Code

```
Queue q = new Queue();  
q.enqueue(6);  
q.enqueue(4);  
q.enqueue(7);  
q.enqueue(3);
```



List Queue Example



//Java Code

```
Queue q = new Queue();  
q.enqueue(6);  
q.enqueue(4);  
q.enqueue(7);  
q.enqueue(3);  
q.dequeue();
```

Queue: Circular Linked List Implementation



- Possible implementations of a queue
 - A circular linked list with one external reference
 - A reference to the back

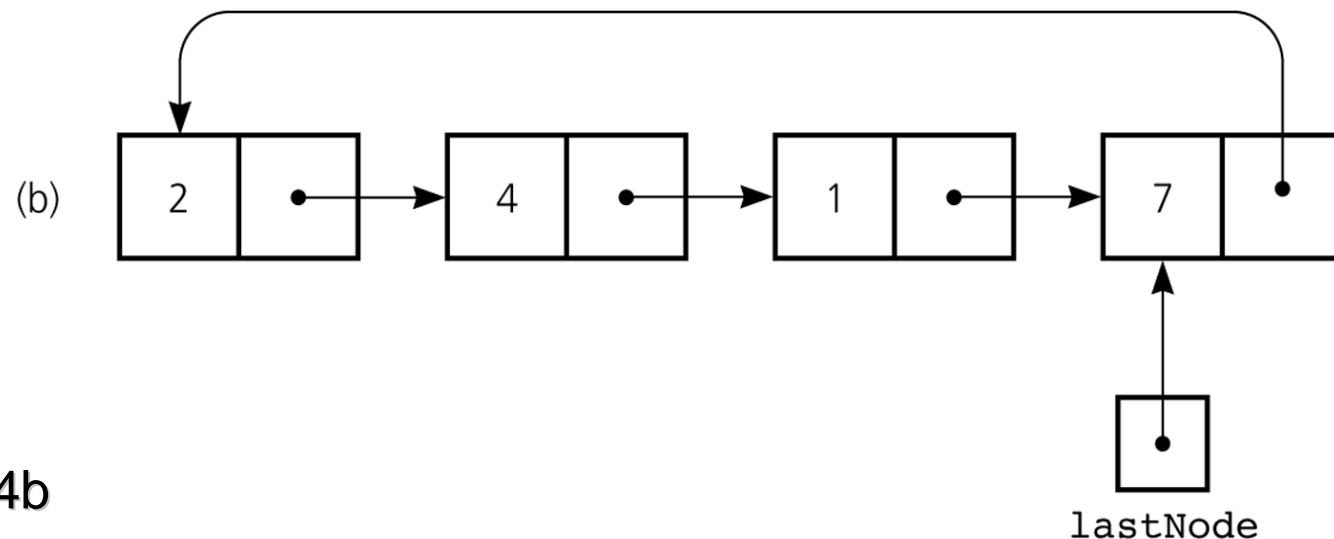


Figure 8-4b

A reference-based implementation of a queue: b) a circular linear linked list with one external reference

Queue: Circular Linked List Implementation

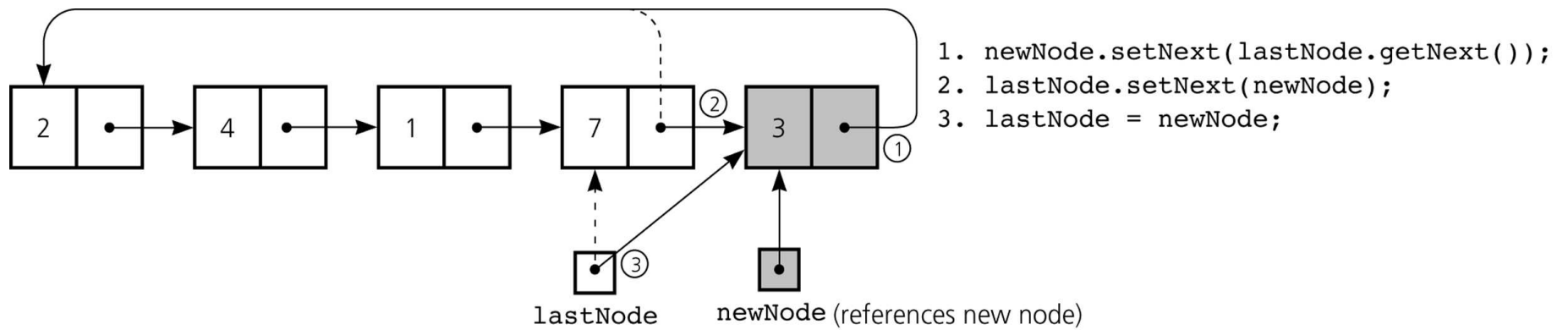


Figure 8-5

Inserting an item into a nonempty queue

Queue: Circular Linked List Implementation

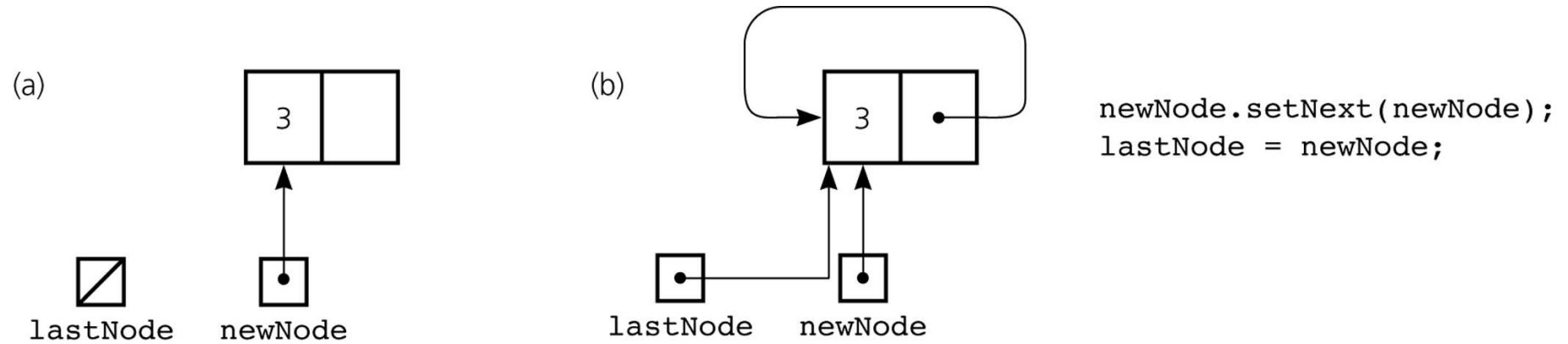


Figure 8-6

Inserting an item into an empty queue: a) before insertion; b) after insertion

Queue: Circular Linked List Implementation

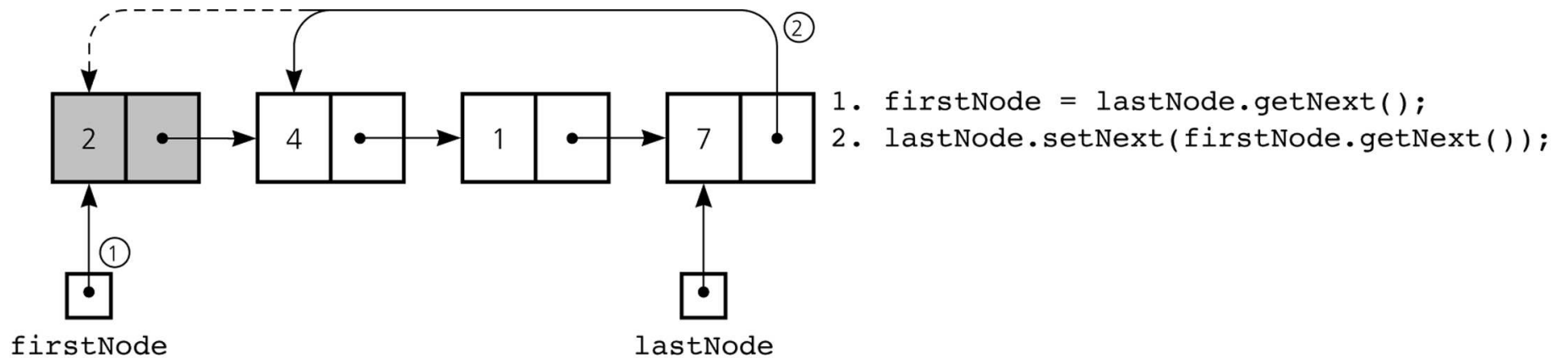


Figure 8-7

Deleting an item from a queue of more than one item

Queue: ADT List Implementation



```
public void enqueue(Object newItem) {  
    list.add(list.size()+1, newItem);  
} // end enqueue  
public Object dequeue() {  
    Object temp = list.get(1);  
    list.remove(1);  
    return temp;  
} // end dequeue
```

Queue: ADT List Implementation



- Efficiency depends on implementation of ADT List – in most common implementations, at least one of operations *enqueue()* and *dequeue()* is not efficient
- On other hand: it was very fast to implement (code is easy, unlikely that errors were introduced when coding).

Application of queues: Recognizing Palindromes



- A palindrome
 - A string of characters that reads the same from left to right as it does from right to left
- To recognize a palindrome, a queue can be used in conjunction with a stack
 - A stack can be used to reverse the order of occurrences
 - A queue can be used to preserve the order of occurrences

Recognizing Palindromes



- A nonrecursive recognition algorithm for palindromes
 - As you traverse the character string from left to right, insert each character into both a queue and a stack
 - Compare the characters at the front of the queue and the top of the stack

String: abcdb

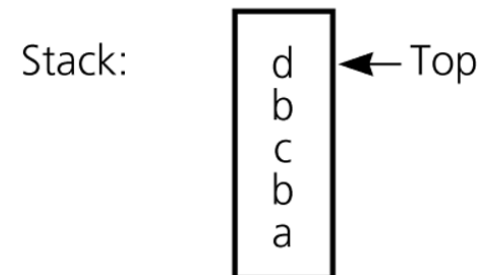
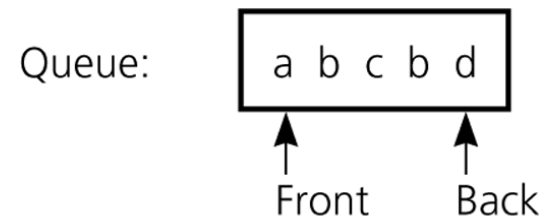
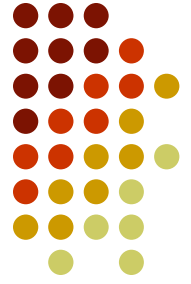


Figure 8-3

The results of inserting a string into both a queue and a stack



Problems:

- Recognize palindromes using 3 stacks.
- Simulate (implement) a queue with 2 stacks.
How efficient is this implementation?
- Simulate (implement) a stack with 1 queue!
(This is the problem which I wanted to discuss but didn't recall it's formulation exactly.) How efficient is this implementation?

Recursion

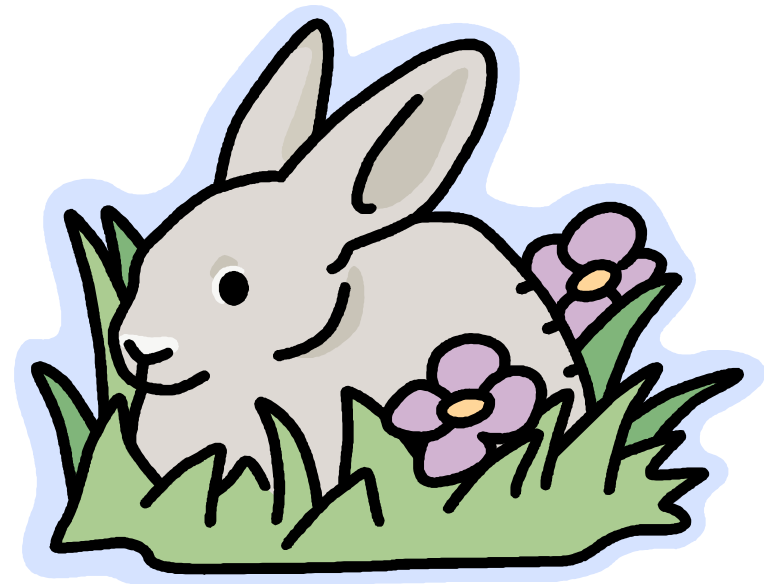


- An extremely powerful problem-solving technique
- Breaks a problem in smaller identical problems
- An alternative to iteration
 - An iterative solution involves loops



Example: Rabbits

- What happens if you put a pair of rabbits in a field?
 - You get more rabbits!
- Assume that rabbits take one month to reach maturity and that
- Each pair of rabbits produces another pair of rabbits one month after mating.
- **That is:** each pair will produce a new pair 2 months after it was born (and every month after that)





Example: Rabbits

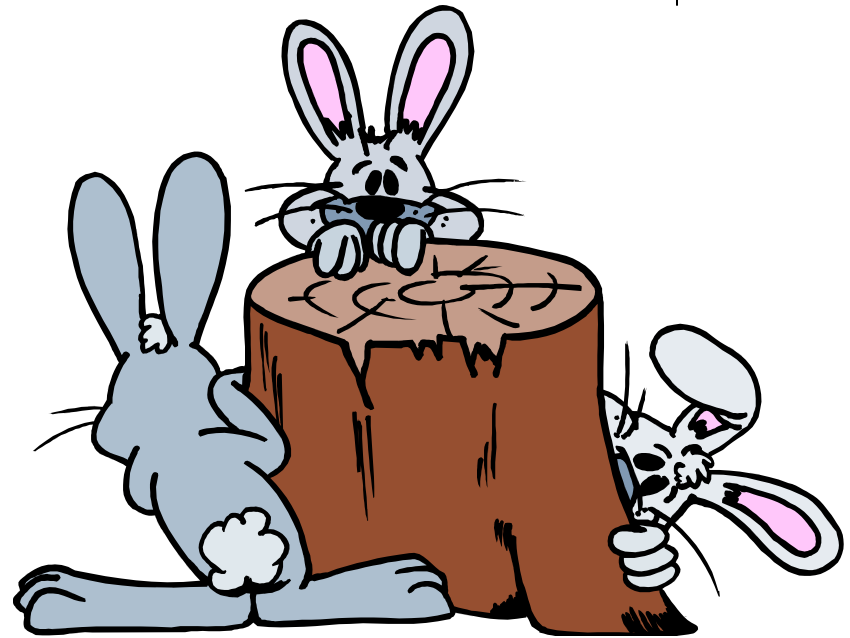
- How many pairs of rabbits are there after five months?
- Month 1: 1 pair
- Month 2: 1 pair
 - after one month the rabbits are mature and can mate
- Month 3: 2 pairs
 - the first pair gives birth to a new pair of rabbits
- Month 4: 3 pairs
 - the first pair gives birth to a new pair, her first children are now mature
- Month 5: 5 pairs





Example: Rabbits

- After 5 months there are 5 pairs of rabbits
 - i.e. the number of pairs at 4 months (3) plus the number of pairs at 3 months (2)
 - Why?
- We have count existing pairs of rabbits (the same number as previous month) + the new pairs (the same number as 2 months ago, as only those rabbits can now produce children)
- This series of numbers is called the Fibonacci series



$$\text{rabbit}(n) = \text{rabbit}(n-1) + \text{rabbit}(n-2)$$

Multiplying Rabbits (The Fibonacci Sequence)

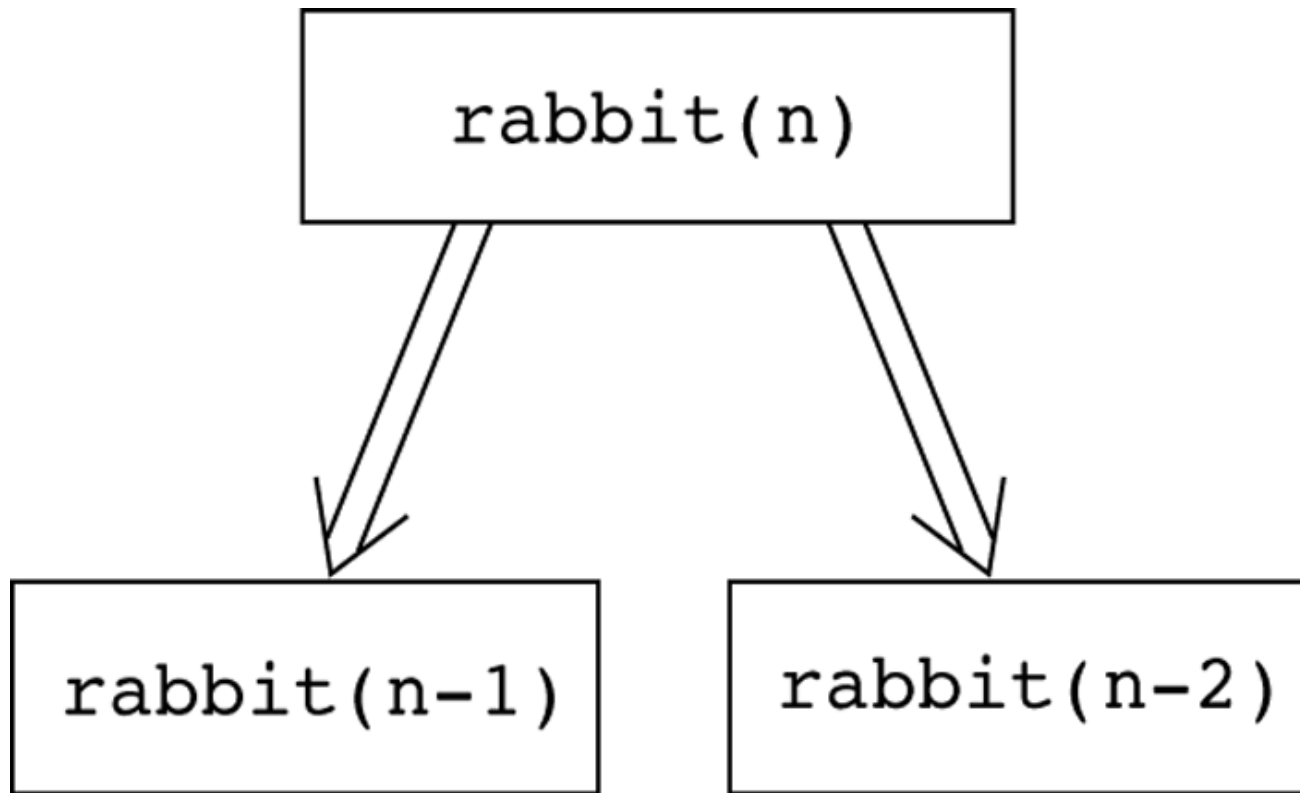


Figure 3-10

Recursive solution to the rabbit problem



Fibonacci Sequence

- The n^{th} number in the Fibonacci sequence, $fib(n)$, is:
 - 0 if $n = 0$, and 1 if $n = 1$
 - $fib(n - 1) + fib(n - 2)$ for $n > 1$
- So what, say, is $fib(23)$, or how many pairs of rabbits would there be after 23 months?
 - This would be easy if only we knew $fib(22)$ and $fib(21)$
 - If we did then the answer is just $fib(22) + fib(21)$
 - What happens if we write a function to calculate Fibonacci numbers in this way?

Calculating the Fibonacci Sequence



- Here is a function to return nth number in the Fibonacci sequence

- e.g. $\text{fib}(1) = 1$, $\text{fib}(3) = 2$, $\text{fib}(5) = 5$, $\text{fib}(8) = 21$, ...

```
public static int fib(int n){
    if(n == 0 || n == 1){
        return n;
    }
    else{
        return fib(n-1) + fib(n-2);
    }
}
```

The function calls itself!!!

- Notice this looks just like the description of the Fibonacci sequence given previously, but does it work!?