

CMPT-225 Jan Manuch

Recommended Labs – Monday, July 24, 2006

The goal of this lab is to get familiar with heaps.

Task 1. Prove that the number of nodes of a complete binary tree is between 2^{h-1} and $2^h - 1$ (inclusive), where h is the height of the tree (the number of nodes on the longest path from root to a leaf). Verify that it follows that $h = 1 + \lfloor \log_2 n \rfloor$.

Task 2. Prove that in a heap, the children of a node with index i , have indices $2i+1$ and $2i+2$.

Hint:

1. first, observe that nodes on level k have indices from $2^{k-1} - 1$ to $2^k - 2$;
2. let k be the level of node with index i , then $i = 2^{k-1} - 1 + j$, where the node is $j+1$ -st node on the level k (counted from left). What is the index of the first node on the next level (level $k+1$)? How many nodes are there before the left child of node with index i on level $k+1$? Express the index of the left child of the node using k and j .