Finite State Machines

Classes

- Formal Languages
- Finite State Machines

Formal Languages

Natural Languages

- A natural language is used for human communication
  - Spoken, written or gestured
    - e.g. English, French, Mandarin, Klingon
- There are rules
  - Valid characters
  - Valid words
  - Valid sentences
  - Acceptable idioms

The human brain is pretty good at coping with language errors

Computers, considerably less so

Formal Languages

- A formal language is used to distinguish precisely what is allowed from what is not
  - Expressed mathematically, often using recursion
    - e.g. valid postfix expressions, valid C++
- Similarly to natural languages there are
  - Alphabets
  - Words
  - Grammars
  - But no idioms

Alphabets and Words

- An *alphabet* is a finite collection of symbols
  - e.g. Σ = {a, b, c, …, x, y, z} – letters of the alphabet
  - e.g. Σ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} – base ten digits
  - e.g. Σ = {0, 1} – binary digits
- A *word* is a finite sequence of alphabet symbols
  - Symbols may be repeated
    - e.g. baa, 100, wool, sheep
  - Order matters
    - e.g. listen, silent
  - The word of length zero is special

Languages

- A (formal) language is a set of words
  - Can be finite
    - e.g. *L* = {all valid English words}
  - Or infinite
    - e.g. *L* = {all valid decimal numbers}
  - But the words themselves are of finite length
- Rules specify which words are valid or invalid
  - The rules that describe a language are referred to as a *grammar*

Specifying a Formal Language

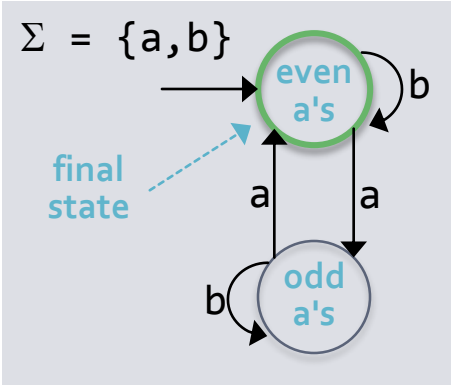
- Like a natural language, use a grammar
  - Describe the symbols allowed and the order in which they should appear
    - Usually specified recursively
- Examples
  - A valid sentence is a noun phrase followed by a verb phrase followed by a subordinate clause
    - A subordinate clause may be composed of the symbol *where* followed by a valid sentence
  - A valid postfix expression is either a single number or two valid postfix expressions followed by an operator

Representing a Grammar

- A grammar can be represented using *production rules*
  - For postfix
    - E $\rightarrow$ number
    - E $\rightarrow$ EE operator
    - A postfix expression is either a number or two postfix expressions followed by an operator
- We can write algorithms to take input, break it into its components and determine if it is syntactically correct
  - Known as a *parser*
  - Parsing is the process of analyzing a string of symbols that conform to a grammar

Modelling Computation

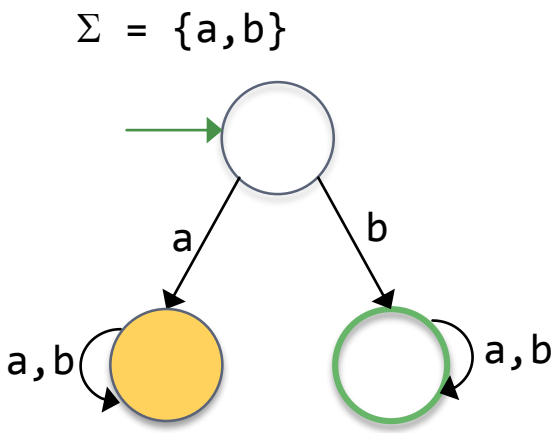
- Use a *finite state machine* to model the rules of a language
- FSM rules
  - Finite number of states
    - FSM reads one character at a time
    - Next state is determined by looking at the current state and the next input character, and nothing else
    - Each state has at most one transition on any given character
    - Previously read characters may not be read again
  - One state is identified as the *start* state
    - One or more states are identified as *final* states
    - If the last state is a final state – accept
    - If the last state is not a final state – reject

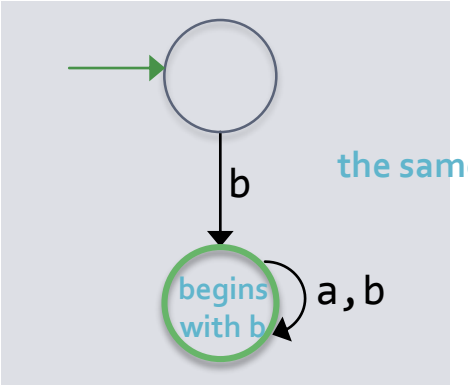


Finite State Machines

Dead States

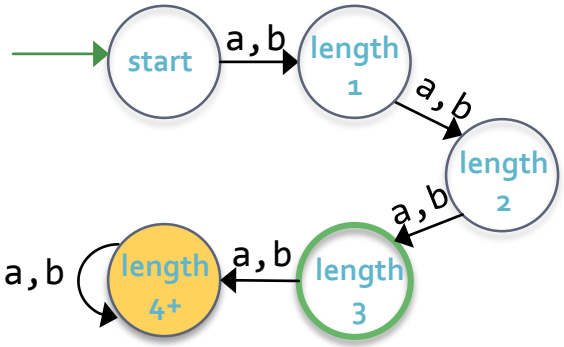
- In this presentation
  - Final states outlined in green
  - Start state pointed to by a green arrow
- The yellow state is a dead state
  - Dead states are not final states
  - Dead states cannot be transitioned from
    - They only transition to themselves
  - Dead states are usually not shown
    - In addition transitions that are not shown go to a dead state

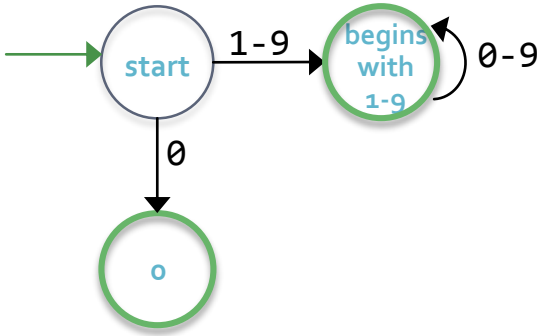




Using the Dead State

- Build an FSM that accepts all words of length 3
  - $\Sigma = \{a, b\}$
- Build an FSM that accepts all decimal integers
  - Disallow leading zeros
  - $\Sigma = \{0,1,2,3,4,5,6,7,8,9\}$
  - Dead state not shown
    - Any transition from 0 final state

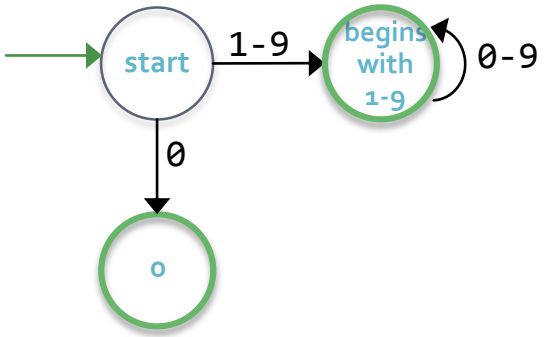




FSM Implementation

- Implement in a simple loop
- Algorithm:

```
state = start
while there is still input
    c = next input symbol
    if transition(state, c) exists
        state = transition(state, c)
    else
        reject (or state = dead state)
end while
if state is a final state accept
else reject
```



Implement transitions with a **table** or a case statement

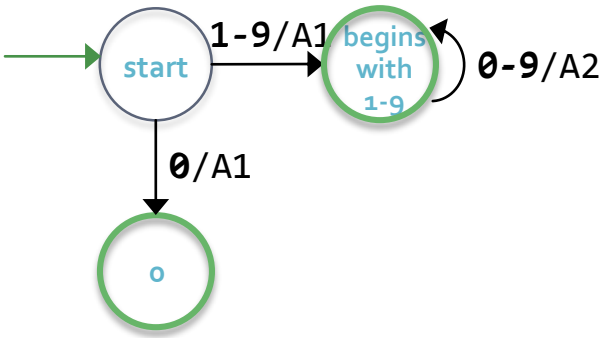
state / c	0	1-9
start	begin 0	begin 1-9
begin 0	dead	dead
begin 1-9	begin 1-9	begin 1-9

Augmenting FSMs

- FSMs can be augmented with other information
- Actions
  - Transitions to a state may be associated with an action
    - Such as the calculation of a value
  - Shown after the input character on the transition arc
    - Typically separated from the input by a /
- Output
  - Transitions may also be associated with output
  - Again, shown after the input character(s) associated with the transition

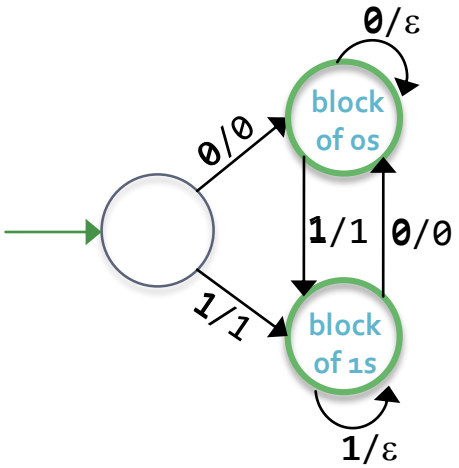
FSM Actions

- Perform an action during a transition
  - Place actions on transition, following a slash
- What might be a useful action in the FSM to accept integers?
  - The value of the integer
    - A1: val = c
    - A2: val = 10 * val + c



FSM Output

- Build an FSM that performs block reduction
  - That reports 0 for each sequence of 0s
  - And 1 for each sequence of 1s
- Example
  - 11100001001110011 goes to
  - 1010101
- Note that ε represents the empty string



FSM Summary

- FSMs are a mathematical model of computation
  - The behavior of many devices can be modeled by a state machine
    - Vending machines
    - Traffic lights
    - …
- FSMs can be used to model systems
  - In engineering
  - And computing science
    - To model the behavior of an application