Droste Effect:
The picture is in the picture which is in the picture …

CMPT 120

Lecture 20 – Graphics and Animation

Python – Implementing and Visualizing **Recursion**

Last Lectures

- Solved the **chocolate chip cookie problem** using **Turtle** + **Loops** + **Functions** + **Tuples**

- Summarized various topics related to **functions** using
  - OperationsOnList.py posted on our course website
  - And the Python Visualizer

- Introduced (very briefly) a new kind of algorithm: **Recursion**

- We had our **Practice Exam 5**: Feedback

Today's Menu

- Investigate **Recursion**
- Solve problems using **recursion**
- **Visualize** the execution of our **recursive** solutions

Review: Recursion - Definition

From our Readings (16.1)

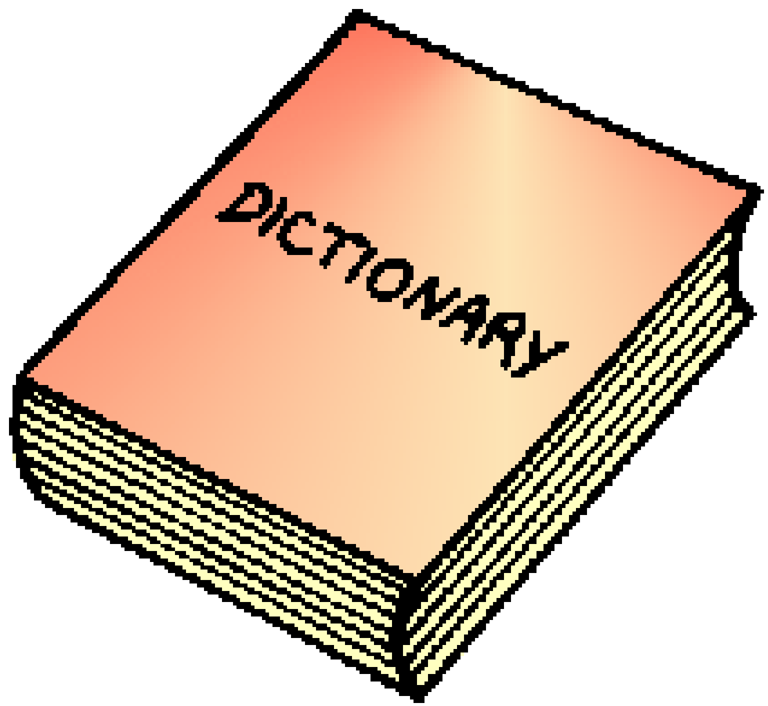
- **Recursion** is a method of solving problems that involves breaking a problem down into smaller and smaller subproblems until you get to a small enough problem that it can be solved trivially.

- **Recursion** occurs when an **object** or a **process** is defined in terms of itself (or a version of itself).

Review: Recursion in the **real world**

- Russian dolls

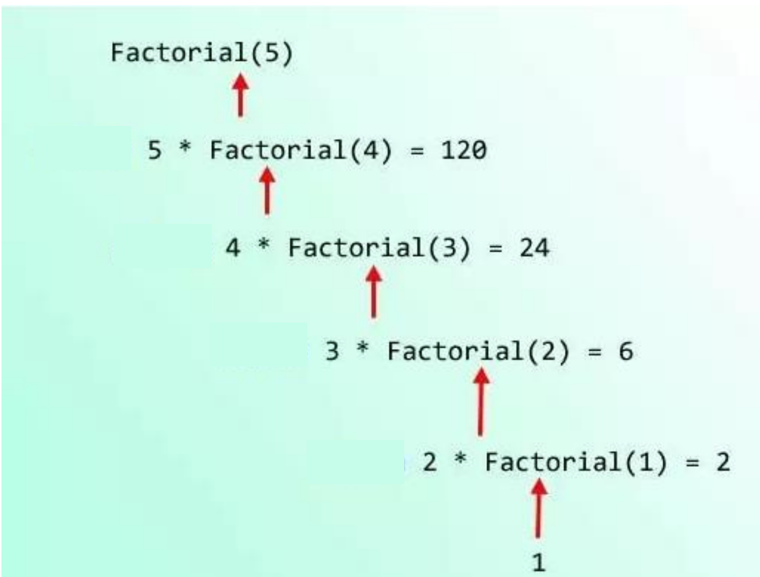
- Searching for a word in a dictionary



Source: http://www.eslstation.net/ESL310L/310L_dict.htm

Recursion in the **mathematical world**

- Factorials

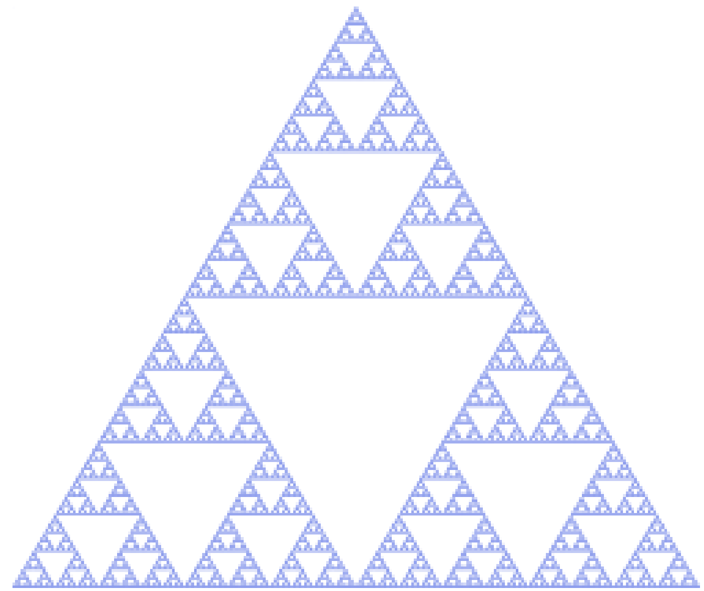


The factorial rule:

- n! = n × (n−1)!   if n > 1
- n! = 1   if n = 1 or 0

Source: https://www.quora.com/How-is-recursion-used-to-compute-the-factorial-function

- Fractals



The Sierpinski triangle
is a confined recursion of triangles
that form a fractal

See section 16.6 in our online e-textbook

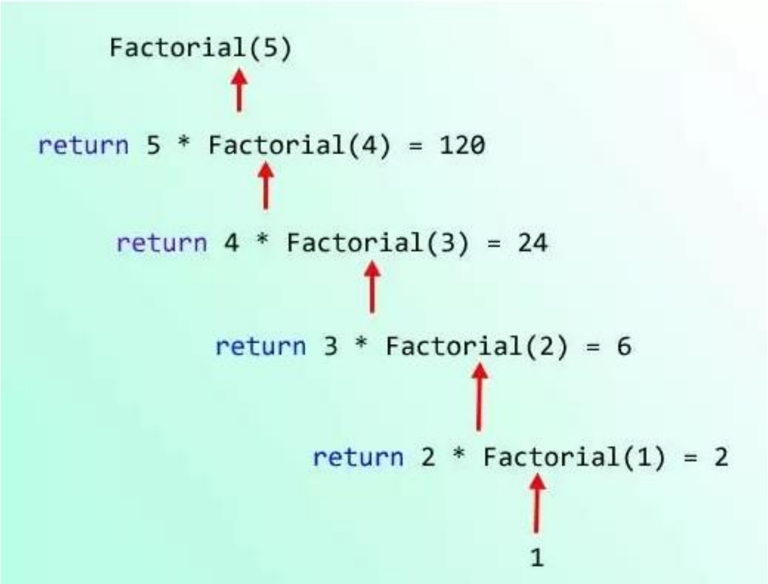
Source: https://en.wikipedia.org/wiki/Recursion

Recursion in the **software world**

- So far, when solving problems using software (algorithms), we have achieved iteration by using **iterative** statements -> **loops**
  - By putting statements we wanted to execute more than once in a **loop**

Recursion in the **software world**

- **Recursion** is an **elegant** way of solving problems where we achieve iteration by putting statements we want to execute more than once in a **function** and having this **function** calling itself



Let's give Recursion a go!

Step 1 - Problem Statement

- Implement a **factorial** function using **recursion**

The factorial rule:

- $n! = n \times (n-1)!$ if $n > 1$
- $n! = 1$ if $n = 1$ or 0

Step 2 – Design?

Step 3 – Implementation

Step 4 - Testing

Source: https://en.wikipedia.org/wiki/File:Man-scratching-head.gif

Step 2 – Design

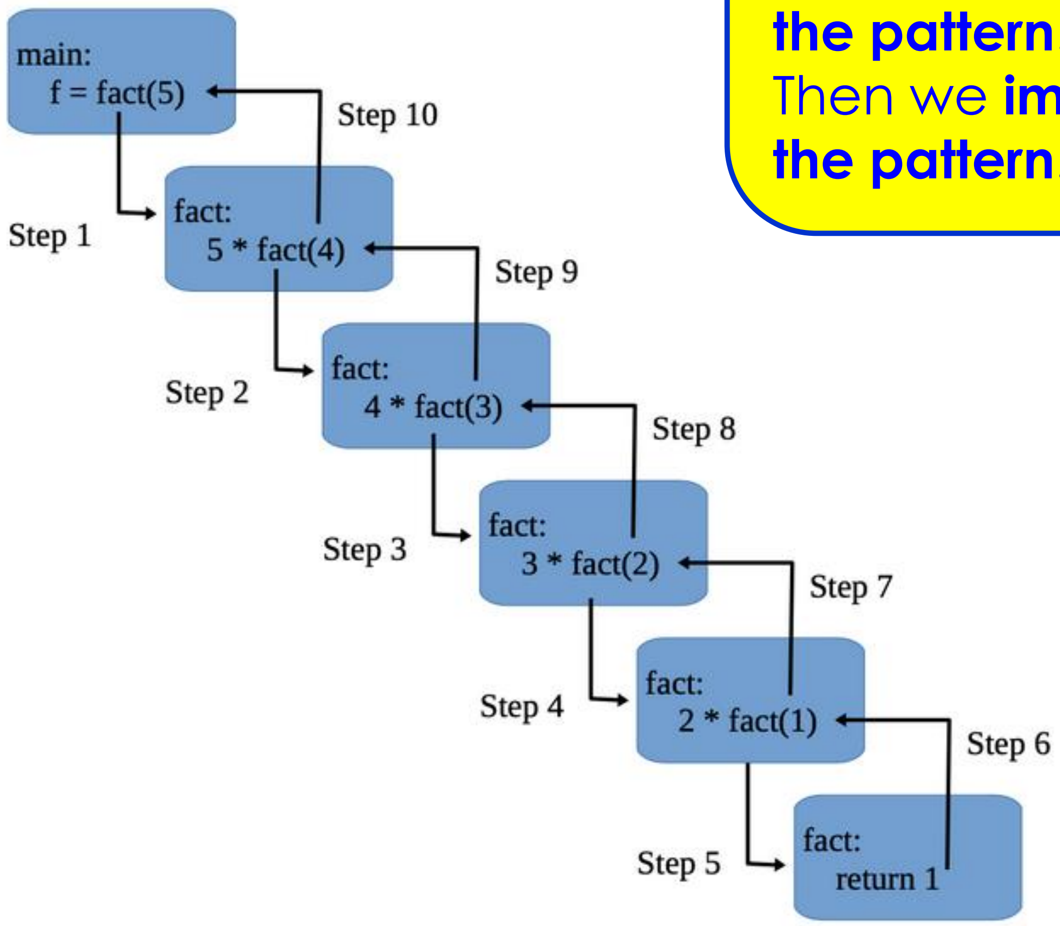
- Design **recursive function** in two parts called ***case***:
  1. **Base case**: Version of the problem that is small enough to be solved trivially
     - **Function** stops calling itself and we start "recursing up".

       The factorial rule:
       - $n! = n \times (n-1)!$ if $n > 1$
       - $n! = 1$ if $n = 1$ or 0
  2. **Recursive case**: Break a problem down into smaller version of itself to eventually become the base case
     - **Function** calling itself with diminishing argument(s)

       The factorial rule:
       - $n! = n \times (n-1)!$ if $n > 1$
       - $n! = 1$ if $n = 1$ or 0

Step 2 – Design

Using an example, we observe how we solve the factorial problem by hand **looking for the pattern**.
Then we **implement the pattern**!



Source: https://eng.libretexts.org/Bookshelves/Computer_Science/Programming_Languages/Introduction_to_Programming_using_Fortran_95_2003_2008_%28Jorgensen%29/18%3A_Recursion/18.05%3A_Recursive_Factorial_Function_Call_Tree

Step 3 – Implementation

And let's **visualize** the execution of our **factorial function** as we are performing **Step 4 – Testing**

Let's try again!

Step 1 - Problem Statement

- Remember the **palindrome function** of Practice Exam 4?
- Solve the **palindrome problem** **recursively**

Step 2 – Design

Using an example, we observe how we solve a palindrome problem by hand, **thinking recursively** and **looking for the pattern**. Then we **implement the pattern**!

Hum... Easy for you to say!

Source: https://en.wikipedia.org/wiki/File:Man-scratching-head.gif

Step 2 - Design

- Using an example: **`kayak`**

Step 3 – Implementation

Step 4 - Testing

Next Lecture

- Use **Computer Graphics**, **turtle** and **recursion** to draw **trees**