MACM 101 (Surrey) Midterm, Spring 2018

Please write your answers in the exam booklet. **Show your work**: answers without explanations won't get full marks!

1. The students in Mrs. Reid's first grade class consist of 10 boys and 15 girls, and they all have different heights.
   (a) (1 point) How many different ways can the students stand in a line?
   (b) (2 points) How many ways can the students stand in a line such that the first person is *not* the tallest or shortest, and the last person is *not* the tallest or shortest?
   (c) (2 points) How many ways can they stand in a line so that all the boys are before all the girls (i.e. no girl is before a boy)?
   (d) (3 points) If the students go on a field trip in 3 buses, how many different ways can they be distributed among the buses? For this question, assume the students are indistinguishable, and the order they sit on the buses doesn't matter.

2. (a) (3 points) Define $\binom{n}{k}$.
   (b) (2 points) Evaluate the following as a single integer (show your work!):

$$\sum_{i=0}^{5} \binom{5}{i}$$

   (c) (5 points) Prove that for any integer $n \geq 2$:

$$\binom{n}{2} = \frac{n(n-1)}{2}$$

3. (5 points) Give a *logically equivalent* expression for $p \to q$ that does *not* use $\to$, and prove that it is logically equivalent to $p \to q$.

4. (5 points) Consider the following argument:

$$\begin{array}{l} \neg B \vee M \\ \underline{\neg(\neg B \wedge G)} \\ \therefore G \vee M \end{array}$$

   Is it valid or invalid? Prove your answer is correct.

5. Suppose $p(n)$ and $q(n)$ are defined as follows:

$$p(n) : n \text{ is a multiple of } 5$$
$$q(n) : n \text{ is a multiple of } 10$$

Assuming the universe of discourse is all integers greater than 0, re-write each of the following English statements as logically equivalent statements in quantified logic:

(a) (1 point) 3 and 25 are multiples of 5, but 7 isn't.

(b) (2 points) Every multiple of 10 is a multiple of 5.

(c) (2 points) Some multiples of 5 are not multiples of 10.