# MACM 101 (Surrey) Midterm Review Questions, Spring 2018

1. In 10-pin bowling, the goal is to knock down 10 pins, numbered 1 to 10. You get to roll two balls, one after the other. If you knock down all 10 on your first ball, that's called a *strike*. If you don't get a strike, how many different patterns of pins are possible after the first ball?

2. Recall that a *bit* is a 0 or a 1.
   (a) An *IPv4* Internet address has the form $W.X.Y.Z$, where $W$, $X$, $Y$, and $Z$ are each integers from 0 to 255. For example, 142.58.102.68 is the IPv4 address for `www.sfu.ca`. How many different IPv4 addresses are possible?
   (b) How many bits are needed to represent one *IPv4* address?
   (c) An *IPv6* Internet address consists of 16 *octets*, where one octet is 8 bits. How many different IPv6 addresses are possible?

3. Prove that the following equality is true for all valid integers $n$ and $k$:
$$\binom{n}{k} = \binom{n}{n-k}$$

4. State the *binomial theorem*.

5. An ice cream parlor sells 31 different flavors of ice cream. How many different ways can you select 3 scoops of ice cream where i) the order matters, and ii) the order doesn't matter?

6. State the *Modus Tollens* inference rule, and prove that it's valid.

7. Show that the following rule of inference is *not* valid.
$$\begin{array}{l} \neg p \\ \underline{p \to q} \\ \therefore \neg q \end{array}$$

8. Suppose $p(x, y)$ means "$xy = 0$", and the universe of discourse is all real numbers. You may also use statements of the form "$a = b$", where $a$ and $b$ are numbers are variables. Translate each of the following statements in quantified logic, and also say whether the statement is true, false, or possibly true or false.

(a) 3 times 5 is 0

(b) Either 3 times 5 is 0, or 6 times 0 is not 0.

(c) $x$ times $y$ is 0 if, and only if, $y$ times $x$ is 0.

(d) If $xy = 0$, then either $x$ or $y$ is 0.

(e) If $x$ or $y$ is 0, then so is $xy$.

(f) A negative number times a positive number is never 0.

(g) 0 times any number is 0.

(h) There's a number $x$ such that no number times $x$ is 0.

(i) For every number $x$, there's *exactly one* number that you can multiply $x$ by to get 0.

9. Suppose $E(x, y)$ means "$x = y$", $G(x, y)$ means "$x > y$", and the universe of discourse is all real numbers. Translate each of the following statements into quantified logic, and also say whether the statement is true, false, or possibly true or false.

   (a) 5 is bigger than 2

   (b) All numbers are equal to themselves.

   (c) No number is greater than itself.

   (d) For any two different numbers, one is bigger than the other.

   (e) $x$ equals $y$ if, and only if, $y$ equals $x$.

   (f) If $x$ equals $y$ and $y$ equals $z$, then $x$ equals $z$.

   (g) There's no biggest number.

   (h) There's a biggest number.