MACM 101 (Surrey) Midterm, Fall 2018

Please write your answers in the exam booklet. **Show your work**: answers without explanations won't get full marks! This exam has **6** questions and is out of **40** marks (there are no bonus marks).

1. In a regular deck of 52 cards, 26 are red and 26 are black. Also, there are 4 suits of 13 cards each: spades, hearts, diamonds, and clubs.

   (a) (1 point) How many different ways can all 52 different cards be arranged in a line?

   (b) (2 points) How many different ways can all 52 cards be arranged in a line, assuming all that matters is the suit of the card?

   (c) (2 points) How many different ways can all 52 cards be arranged in a line if all the red cards come first, followed by all the black cards?

   (d) (3 points) How many different ways can all 52 cards be distributed among 4 players so that each gets 5 or more cards? They don't all need to get the same number of cards, and the order of the cards in their hand doesn't matter.

2. (a) (3 points) Define $\binom{n}{k}$ for all values of n and k that make sense.

   (b) (1 point) In the expansion of $(a + b)^{100}$, what is the coefficient of the term $a^{60}b^{40}$?

   (c) (5 points) Prove that this equation holds for all non-negative integers n:

   $$n! + (n+1)! = \frac{(n+2)!}{n+1}$$

3. (5 points) Give a short logical expression that is *logically equivalent* to $p \vee q$ that does *not* use $\vee$. Prove your expression is logically equivalent to $p \vee q$.

4. (5 points) Show that this argument is *invalid*:

$$\neg p \vee q$$
$$\underline{\neg p \vee r}$$
$$\therefore q \vee r$$

5. Suppose $E(n)$ and $O(n)$ are defined as follows:

$$E(n) : n \text{ is exciting}$$
$$O(n) : n \text{ is obvious}$$

Assuming the universe of discourse is all integers, re-write each of the following English statements as logically equivalent statements:

(a) (2 points) 4 is neither exciting or obvious.

(b) (2 points) A number is exciting if, and only if, it's not obvious.

(c) (2 points) No number is both obvious and exciting.

(d) (2 points) Numbers are obvious, unless they're exciting.

6. Assuming only logical statements with a single variable, state the rule of

(a) (2 points) *universal specification.*

(b) (3 points) *universal generalization.*