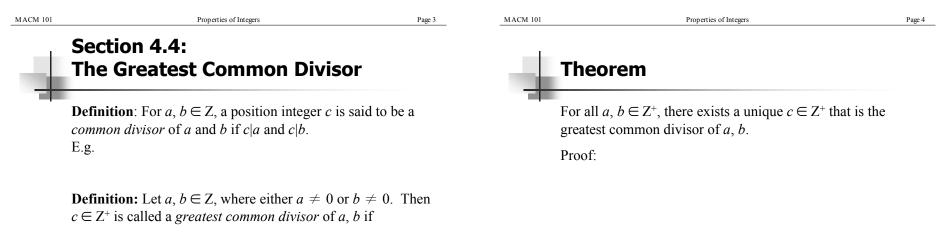
Page 2

- If $a, b \in \mathbb{Z}$, with b > 0, then there exist unique $q, r \in \mathbb{Z}$ with $a = qb + r, 0 \le r < b$.
- > q is referred to as the quotient
- > *r* the remainder
- > *b* is the divisor
- > a is the dividend



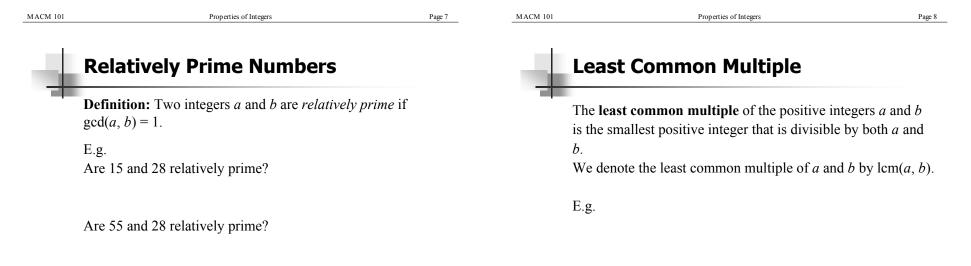
- (a) When a = 170 and b = 11:
- (b) When a = 98 and b = 7:
- (c) When a = -45 and b = 8:



(a) c|a and c|b (that is, c is a common divisor of a, b), and (b) For any common divisor d of a and b, we have d|c

E.g.





Are 35 and 28 relatively prime?

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MACM 101

Section 4.5: The Fundamental Theorem of Arithmetic

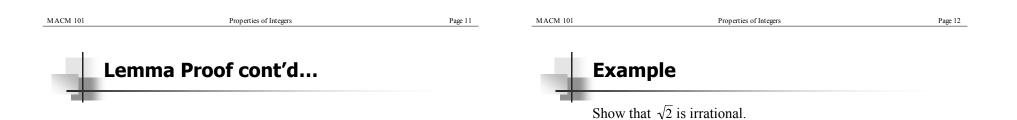
Lemma 4.2: If $a, b \in Z^+$ and p is prime, then $p|ab \Rightarrow p|a \text{ or } p|b.$

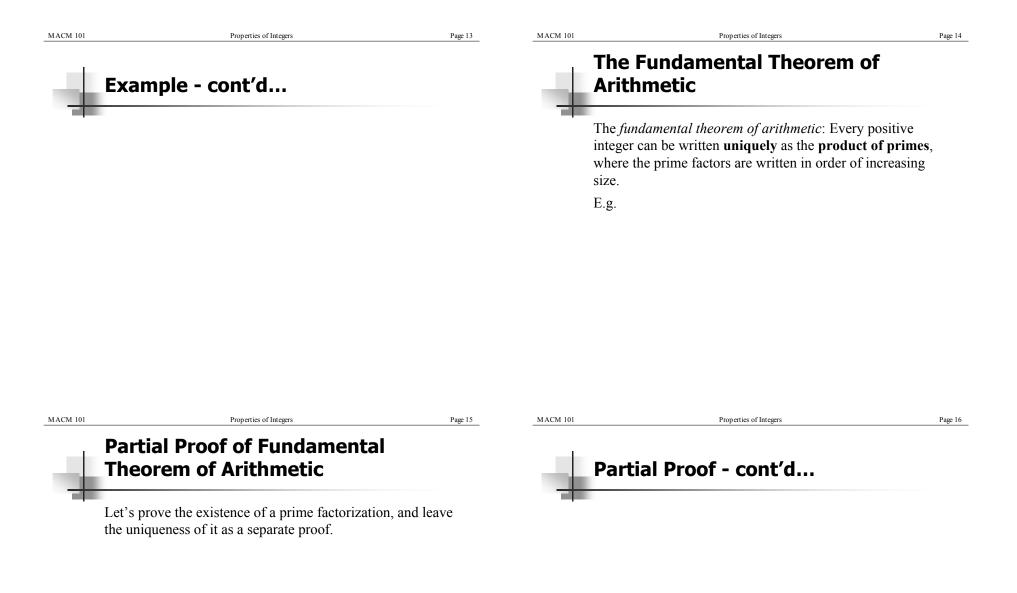
Proof:

Another Lemma

Lemma 4.3: Let $a_i \in \mathbb{Z}^+$ for all $1 \le i \le n$. If *p* is prime and $p|a_1a_2\cdots a_n$, then $p|a_i$ for some $1 \le i \le n$.

Proof:





Properties of Integers

MACM 101

Suppose that $n \mathbb{Z}^+$ and that

 $10.9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot n = 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14$

Show that 17|n

Example

Aside: Pi Notation

We've already seen sigma notation for summations, Pinotation is the same type of notation for multiplications.

E.g. $\prod_{i=1}^{6} x_i =$ $\prod_{i=3}^{6} i =$ $\prod_{i=m}^{n} i =$