

Review — MACM 101 (1091)

- Prove by induction that for any natural n

$$1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \dots + n \cdot (n + 2) = \frac{n(n + 1)(2n + 7)}{6}.$$

- Find the greatest common divisor of 408 and 525. Find u and v such that $\gcd(408, 525) = 408u + 525v$.
- Find the prime factorization of 899.
- Express by a regular expression and finite automata:
 - the set of strings with one or more 0s followed by 1,
 - the set of strings of two or more symbols followed by three or more 0s,
 - the set of strings of 1s so that the number of 1s equals 2 modulo 3, followed by an even number of zeros.
- Find a finite-state automaton that recognizes $\{\lambda, 0\}, \{0, 11\}, \{0, 11, 000\}$.
- What is probability that in a family of 5 children there are two girls? All children are girls?
- Give an example of a binary tree in real life.
- Let A be an automaton over alphabet Σ with n states. We define a relation R over Σ^* as the set of all pairs (x, y) such that automaton A terminates at the same state after application of x and after application of y . Prove that R is an equivalence relation. Prove that each equivalence class of R is a regular language.