Chapter 16

Rational preferences
Utilities
Money
Value of information

Preferences
An agent chooses among prizes (A, B, etc.) and lotteries, i.e., situations with uncertain prizes.

Lottery \( L = \left( p, A; (1-p), B \right) \)

Notation:
\( A \succ B \) A preferred to B
\( A \sim B \) indifference between A and B
\( A \not\succ \sim B \) B not preferred to A

Rational preferences
Idea: preferences of a rational agent must obey constraints.
Rational preferences \( \Rightarrow \) behavior describable as maximization of expected utility

Constraints:
Orderability:
\( A \succ B \lor B \succ A \lor A \sim B \)

Transitivity:
\( A \succ B \land B \succ C \Rightarrow A \succ C \)

Continuity:
\( A \succ B \succ C \Rightarrow \exists p \left[ p, A; 1-p, C \right] \sim B \)

Substitutability:
\( A \sim B \Rightarrow \left[ p, A; 1-p, C \right] \sim \left[ p, B; 1-p, C \right] \)

Monotonicity:
\( A \succ B \Rightarrow \left[ p, A; 1-p, B \right] \succ \sim \left[ q, A; 1-q, B \right] \)

Maximizing expected utility
Theorem (Ramsey, 1931; von Neumann and Morgenstern, 1944):
Given preferences satisfying the constraints, there exists a real-valued function \( U \) such that

\( U(A) \geq U(B) \iff A \succ \sim B \)

\( U(\left[ p, S_1; \ldots; p_n, S_n \right]) = \sum_i p_i U(S_i) \)

MEU principle: Choose the action that maximizes expected utility.

Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities. Even preferences satisfying the constraints may not be consistent with rational decision-making.

An agent chooses among prizes (A, B, C, etc.) and lotteries. Let's consider the following example:

- If B \succ C, an agent who has C would pay (say) 1 cent to get B.
- If A \succ B, an agent who has B would pay (say) 1 cent to get A.
- If C \succ A, an agent who has A would pay (say) 1 cent to get C.

Preferences
Rational preferences contd.
Violating the constraints leads to self-evident irrationality. For example, an agent with intransitive preferences can be induced to give away all its money.

Value of information
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Rational preferences

Outline
Utilities

Utilities map states to real numbers. Which numbers?

Standard approach to assessment of human utilities:

Given a state $A$ compare it to a standard lottery $L_p$ that has "best possible prize" $u^\top$ with probability $p$ and "worst possible catastrophe" $u^\bot$ with probability $(1-p)$. Adjust lottery probability $p$ until $A \sim L_p$.

Student group utility

For each $x$, adjust $p$ until half the class votes for lottery (M=10,000).
A decision problem is often a real-life scenario where a decision maker is faced with a problem that can be broken down into a series of decisions. The decision maker must weigh the possible outcomes of each decision and make a choice that maximizes their expected utility. This is often done by computing the expected value of each possible decision and choosing the one with the highest expected value.

In a decision tree, the decision maker starts at the root node and branches out to represent each possible decision. The branches then lead to various outcomes that represent the possible states of the world given the decision made. The decision maker then assigns probabilities to each outcome and computes the expected value of each decision.

Value of Information: The value of information is the expected increase in the utility of a decision maker due to acquiring additional information. It is often calculated by comparing the expected value of a decision with and without the information.

Nonadditive—expected utility with respect to one piece of evidence can be calculated:

\[
V_{\text{PI}} = \left( E|\text{A}, E \right) - \left( E|\text{A}, \neg E \right)
\]

Value of Perfect Information is the value of acquiring the best piece of evidence.

\[
\text{VPI} = \text{E}(\text{value of best action given information}) - E(\text{value of best action without information})
\]

E.g., a restaurant manager faces the decision to open a new restaurant. He must compute expected gain over all possible values. Suppose we knew whether or not there was a market for his restaurant.

Value of Information:

\[
\text{VPI} = \text{E}(\text{value of best action given information}) - E(\text{value of best action without information})
\]

Expected value of best action given the information

\[
E(\text{Best Action Given Information})
\]

Expected value of best action without information

\[
E(\text{Best Action Without Information})
\]

Example: A used-car buyer is deciding whether to buy car c. q1 is the test of car c. Assume: q1 has a 70% chance of being in good shape. Two blocks and its market value is $2000 if it is in good shape; if not, $700 in repairs.

Fair price?

\[
\text{VPI} = \text{E}(\text{value of best action given information}) - E(\text{value of best action without information})
\]

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