1. Write the following set in set-builder notation.
   (a) \{0, 1, 4, 9, \ldots\}
       \\{x^2 : x \in \mathbb{Z}\}
   (b) \{3, 4, 5, 6, 7, 8\}
       \\{x : -3 \leq x \leq 8\}
   (c) \{-3, -2, -1, 0, 1, 2, 3\}
       \\{x : -3 \leq x \leq 3 \text{ and } x \neq 1\}

2. Suppose \(A = \{0, 1\}\) and \(B = \{1, 2\}\). Find
   • \(\mathcal{P}(A) \cap \mathcal{P}(B)\).
     \{\emptyset, \{1\}\}
   • \(\mathcal{P}(A \cap B)\).
     \{\emptyset, \{1\}\}
   • \(\mathcal{P}(A) - \mathcal{P}(B)\).
     \{\{0\}, \{1, 0\}\}

3. Let \(A, B\) and \(C\) be three arbitrary subsets of the universal set \(U\). Use an element containment proof (i.e. prove that the left side is a subset of the right side and the right side is a subset of the left side) to prove the following:
   • \(\overline{A \cup B} = \overline{A} \cap \overline{B}\).
     Let \(x \in \overline{A \cup B}\)
     \(\Rightarrow x \notin A \cup B\)
     \(\Rightarrow x \notin A \text{ and } x \notin B\)
     \(\Rightarrow x \in \overline{A} \text{ and } x \in \overline{B}\)
     \(\Rightarrow x \in \overline{A} \cap \overline{B}\)
     So \(\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}\).
     Let \(x \in \overline{A} \cap \overline{B}\)
     \(\Rightarrow x \in \overline{A} \text{ and } x \in \overline{B}\)
     \(\Rightarrow x \notin A \text{ or } x \notin B\)
     \(\Rightarrow x \notin A \cup B\)
     \(\Rightarrow x \in \overline{A \cup B}\)
     So \(\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}\)
     Since \(\overline{A} \cap \overline{B}\) and \(\overline{A \cup B}\) are both subsets of one another, so \(\overline{A \cup B} = \overline{A} \cap \overline{B}\).
   • \(\overline{A \cup B \cup C} = \overline{A} \cap B \cap \overline{C}\).
     Proved in the same way as above.

4. Use the membership table method to determine which membership \(\subseteq, =, \supseteq\) is true for the following pair of sets.
\( (A - B) \cup (A - C), \quad A - (B \cap C). \)

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<th>A</th>
<th>B</th>
<th>C</th>
<th>(A-B)</th>
<th>(B-C)</th>
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<th>A- (B \cap C)</th>
<th>(A-B) \cup (A-C)</th>
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From above, we see that \( A-(B \cap C) \) and \( (A - C) \cup (B - C) \) are equal.

5. Two fair six-sided dice are rolled and the sum \( s \) of the numbers coming up is recorded. What is the probability of \( s \geq 10 \)? Show your work.

Let \( S \) be the sample space of the experiment of throwing two dice (red and blue). Then \( S = \{(1,1),(1,2),\ldots,(1,6),(2,1),(2,2),\ldots,(2,6),\ldots,(6,11),(6,2),\ldots,(6,6)\} \). The cardinality of \( S \) is 36. The probability of an outcome is \( \frac{1}{36} \). Let \( A \subseteq S \) where \( A = \{(a,b) \in S|a+b \geq 10\} \). Hence \( A = \{(4,6),(5,5),(6,4)\} \).

Therefore, \( Pr(A) = \frac{|A|}{|S|} = \frac{3}{36} \).

6. A random experiment consists of rolling an unfair, six-sided die. The digit 6 is three times as likely to appear as the numbers 2 and 4. The numbers 2 and 4 are twice as likely to appear as one of the numbers, 1, 3, and 5.

Assign appropriate probabilities to the six outcomes in the sample space.

Since the die is unfair, the probability of the outcomes are not the same. Let \( p \) be the probability of each of the outcomes of 1, 3 and 5. Then the probability of each of the outcomes of 2 and 4 is \( 2p \). The the probability of outcome 6 is \( 6p \).

Since \( 6p + 4p + p + p + p = 1 \), therefore \( p = \frac{1}{13} \). Therefore, \( Pr\{\{1\}\}Pr\{\{3\}\} = Pr\{\{5\}\} = 1/13 \). \( Pr\{\{2\}\} = Pr\{\{4\}\} = \frac{2}{13} \). Lastly, \( Pr\{\{6\}\} = \frac{6}{13} \).