October 15 Tutorial Problems

Quantified propositions → English statements.

1. **Question 6, section 1.4 of Rosen**
   Let $N(x)$ be the statement “$x$ has visited North Dakota” where the domain consists of the students in your school. Express each of these quantifications in English.
   a) $\exists x \ N(x)$
   b) $\forall x \ N(x)$
   c) $\neg \exists x \ N(x)$
   d) $\exists x \ \neg N(x)$
   e) $\neg \forall x \ N(x)$
   f) $\forall x \ \neg N(x)$

English statements → quantified propositions.

1. **Question 10, section 1.4 of Rosen.**
   Let $C(x)$ be the statement “$x$ has a cat,” let $D(x)$ be the statement “$x$ has a dog,” and let $F(x)$ be the statement “$x$ has a ferret.” Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.
   a) A student in your class has a cat, a dog, and a ferret.
   b) All students in your class have a cat, a dog, and a ferret.
   c) Some student in your class has a cat and a ferret, but not a dog.
   d) No student in your class has a cat, a dog, and a ferret.
   e) For each of the three animals, cats, dogs and ferrets, there is a student in your class who has this animal as a pet.

2. **Question 42, section 1.4 of Rosen.**
   Express each of these system specifications using predicates, quantifiers, and logical connectives.
   a) Every user has access to an electronic mailbox.
   b) The system mailbox can be accessed by everyone in the group if the file system is locked.
   c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.
   d) At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy service is not in diagnostic mode.

Contradicting quantified propositions.

1. **Question 36, section 1.4 of Rosen.**
   Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.
   a) $\forall x \ (x^2 \neq x)$
   b) $\forall x \ (x^2 \neq 2)$
   c) $\forall x \ (|x| > 0)$