MACM 101 : Tutorial (November 5, 2018)

1. Answer the following short questions.

   (a) Give an example of a function \( f \) from integers to integers (\( f : \mathbb{Z} \to \mathbb{Z} \)) that is onto but not one-to-one.

   (b) Does the formula \( f(x) = \frac{1}{x^2 - 2} \) define a function \( f : \mathbb{R} \to \mathbb{R} \)? A function \( f : \mathbb{Z} \to \mathbb{R} \)?

   (c) Show that for any positive integer \( a \), \( \lfloor \frac{a}{2} \rfloor + \lceil \frac{a}{2} \rceil = a \).

   (d) Let \( f : \mathbb{Z}^+ \to \mathbb{Z}^+ \) where for all \( x \in \mathbb{Z}^+ \), \( f(x) = \max\{1, x - 1\} \), the maximum of 1 and \( x - 1 \). What is the range of \( f \)? Is \( f \) one-to-one? Is it onto?

   (e) Prove or disprove: If functions \( f : A \to B \) and \( g : B \to C \) are onto, then \( g \circ f \) is onto.

2. Show that the function \( f : \mathbb{R} - \{3\} \to \mathbb{R} - \{2\} \) defined by \( f(x) = \frac{2x - 3}{x - 3} \) is a bijection, and find the inverse function.

3. Let \( A = \{1, 2, 3, 4, 5\} \) and \( B = \{1, 2, 3, 4, 5, 6\} \). How many one-to-one functions \( f : A \to B \) satisfy (a) \( f(1) = 3 \)? (b) \( f(1) = 3, f(2) = 6 \).