### Practice Problems

1. Exercise Problems from ZYbook: 6.7.1, 6.7.2, 6.9.2, 6.9.3
2. Problems from the text: (page 379) 4, 12, 15, 16 (i,ii,iii), 26.
3. Problems from the text: (page 208) 1(b), 2(b), 4, 14, 24

### Homework Problems

1. Let $A = \{6 : 00, 6 : 30, 7 : 00, 7 : 30, \ldots, 9 : 30, 10 : 00\}$ denote the set of nine half-hour periods in the evening. Let $B = \{3, 12, 15, 17\}$ denote the set of four local television channels. Let $R_1$ and $R_2$ be two relations from $A$ to $B$, $R_1, R_2 \subseteq A \times B$. Give examples of $R_1$ and $R_2$ such that $|R_1| = 4$ and $|R_2| = 4$ and $|R_1 - R_2| = 1$.

2. Let $I$ be the set of integers from 1 to 7.
   - (a) Is there a natural way to interpret the ordered pairs in $I \times I$ as points in the plane?
   - (b) What are the elements of the relation $R \subseteq I \times I$ defined by $xRy$ iff $x \leq y$? Give a matrix representation of the relation $R$.
   - (c) Let $R'$ be a binary relation on $I \times I$, i.e. $R' \subseteq (I \times I) \times (I \times I)$.
     - Write an element of $R'$.
     - What is the cardinality of the set $(I \times I) \times (I \times I)$?

3. If $B = \{a, b, c, d, e\}$ and $R$ is an equivalence relation on $B$ which has the partition $\{a, b, c\}$ and $\{d, e\}$. Draw the directed graph representing $R$.

4. Relation $R$ is given by the following matrix:

   \[
   \begin{pmatrix}
   1 & 0 & 0 & 0 \\
   0 & 1 & 0 & 0 \\
   1 & 1 & 1 & 0 \\
   1 & 1 & 1 & 1 \\
   \end{pmatrix}
   \]

   Is $R$ a partial order? If yes, justify; what its minimal, maximal, least, and greatest elements are; how does the Hasse diagram look like?

5. Let $R$ be a relation on the set of propositions such that $R = \{(p, q)|p \leftrightarrow q \text{ is true}\}$. Determine whether $R$ is reflexive, symmetric, antisymmetric, transitive, partial order, and equivalence relation. Just Yes/No answer is fine.

6. For $|A| = 5$, how many relations $R$ on $A$ are there? How many of these relations are symmetric?

7. The Fibonacci sequence is defined as the sequence starting with $F_1 = 1$ and $F_2 = 1$ and then for $n \geq 3$, $F_n = F_{n-1} + F_{n-2}$.
   - (a) Determine the values of $F_3, F_4, F_5, F_6$ and $F_7$. 

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(b) Prove by induction that $F_1 + F_2 + \ldots + F_n = F_{n+2} - 1$.

8. Prove by induction the following generalization of De Morgan's law to $n \geq 1$ sets:

$$A_1 \cup A_2 \cup \ldots \cup A_n = \overline{A_1 \cap A_2 \cap \ldots \cap A_n}.$$  

9. If $n = 2^t - 1$ for some $t \in \mathbb{N}$, then every entry in the $n^{th}$ row of Pascal's triangle is odd.

The following figure indicates the coefficients for some initial rows of Pascal's triangle (each 'o' indicates an odd number and 'E' indicates an even number):

![Figure 1](image)

The starting row number in the figure is 0 (zero).