ON IMPROVING THE ACCURACY OF LINE EXTRACTION IN HOUGH SPACE

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This paper presents an accurate line extraction technique—the Hierarchical Peak Compaction Hough Transform (HPCHT). Vote scattering in the parameter space is a problem when the Hough transform is used for line extraction. This paper investigates the effects of image size and edge data errors on the severity of vote scattering. The HPCHT uses the Hough procedure on small subimages initially, and a recursive Hough merging scheme on the extracted line segments afterwards. A bound on vote scattering has been derived which guides the image subdivision and the adaptive quantization of the parameter space. As a result, an accurate Hough transform of low $\rho$-scattering and high $\theta$-precision has been achieved. The HPCHT is suitable for fast parallel implementation on pyramid computers.

Keywords: Hough transform, adaptive quantization, line extraction, pyramid.

1. INTRODUCTION

The Hough transform has been recognized as a powerful technique for image shape analysis since it was first introduced as a method of detecting patterns in image data.\(^1,2\) The essence of the method lies in the transformation of an image to a parameter space in such a way that spatially extended image patterns are mapped to spatially compact peaks in the parameter space. One of the line extraction methods using the Hough transform maps the edge points in the $x-y$ image space to a $\theta-\rho$ parameter space so that collinear edge points vote to the same peak in the $\theta-\rho$ space. An extensive survey of Hough techniques was presented by Illingworth and Kittler.\(^3\)

The performance of the Hough transform can be improved by using gradient information. The transform from the $x-y$ to the $\theta-\rho$ space is readily reduced to a one-to-one mapping and irrelevant votes arising from random alignment can be eliminated by taking into account the gradient directions.\(^4,5\) Nevertheless, detecting peaks in the parameter space is not trivial because the vote peaks, subject to errors from various sources, are normally blurred and distorted. A great deal of effort has been made to solve the peak detection problem.

Cohen and Toussaint\(^6\) detected the presence of collinear points lying on a noisy background. The density of counts produced in the parameter space by the background noise was equalized by using an equiprobable quantization of the $\rho$-axis (maximum entropy quantization). Non-uniform parameter quantization was further investigated by Alagar and Thiel,\(^7\) leading to a non-uniform quantization which is related to a beta
distribution. The difficulty of the non-uniform quantization approach to noise compensation led Maître\textsuperscript{8} to a signal processing approach where the signal-to-noise ratio was used for performance prediction in adapting the parameters of the Hough process. Rather than quantizing for the background noise, Van Veen and Groen\textsuperscript{9} studied the quantization based on the vote scattering due to discretization of the image and parameter spaces.

Other researchers looked into strategies of vote accumulation and peak interpretation in the Hough process. Van Veen and Groen's gradient weighted Hough transform\textsuperscript{9} spread the votes along the parametric curve in accordance with the probability density function of the noisy gradient directions. Geric and Klein\textsuperscript{10} performed backmapping to the feature space to confirm the detected peaks. At each time, votes are reaccumulated on the remaining feature points for the next most prominent peak. Given the distribution models of the corrupting noise and the image lines, Hunt and Nolte\textsuperscript{11} defined \textit{a posteriori} probabilities for peak interpretation which performed significantly better than the standard Hough transform. Princen, Ilingworth, and Kittler\textsuperscript{12} defined a set of test efficacies for optimizing the peak detectability. Maximum efficacies were used as criteria for designing the kernel function of the filtered Hough voting for optimal performance. Niblack and Petkovic\textsuperscript{13} achieved a high precision parameter estimation by smoothing the Hough accumulator array, which was then interpolated for peaks to yield sub-bucket precision.

Our work focuses on the issue of vote scattering in the parameter space. We have developed a peak compaction Hough transform in which parameter quantization is adaptive. Instead of spreading the votes, we pack the votes into accumulator bins so that the peak interpretation is straightforward. As parameter quantization is adjusted to the error estimates of the line parameters, vote scattering is contained within the grid size of the parameter space and thus peak compaction is possible.

Shapiro\textsuperscript{14} observed that the variance of the parameter values can be influenced by relocating the point of origin of the image space. Following the same line of reasoning, we partition the image into small regions so that vote scattering for the Hough transform local to each of them is constrained. The line segments extracted locally are then merged recursively in a hierarchical structure.

Research in hierarchical schemes for line detection has been pursued primarily in two directions: (a) hierarchical structuring of the Hough parameter space for efficient top-down search for peaks,\textsuperscript{15–18} (b) image pyramid for bottom-up hierarchical line extraction.\textsuperscript{19–23} Our method belongs to the latter category.

Among the bottom-up methods, Shneier's edge pyramid\textsuperscript{19,20} represents edges in multiple resolutions. Edges at each level are a summary of the ones from below in terms of the magnitude and direction, as well as the intercept and error measures. Hong \textit{et al.}\textsuperscript{21} propose a similar pyramid structure for detecting curves. However, Hong's pyramid is built with a different grouping mechanism and multiple curves are represented at each node. Weiss and Boldt\textsuperscript{22} present a hierarchical grouping algorithm in which edges are linked if their end points are close and their contrasts are similar. The linked edges are replaced with a longer straight line if the approximation is sufficiently good.
Princen, Illingworth, and Kittler's Hierarchical Hough Transform (HHT) represents the recent result in hierarchical line finders. Their method differs from its predecessors in its grouping mechanism and representation of the segments. In their method, short line segments are detected using Hough transform at the bottom level of the pyramid. Proceeding bottom-up, their algorithm obtains descriptions of long lines by grouping segments in the Hough parameter space. The grouping mechanism is based on the relationship between the line length and the resolution of the representation. The use of Hough transform, rather than pairwise comparison, is an efficient and natural way to incorporate the resolution criteria. The hierarchical grouping strategy also enforces the proximity constraint implicitly. Moreover, the HHT shows a good accumulation efficiency compared to the standard Hough transform. We adopted a hierarchical line extraction algorithm very similar to the HHT scheme. In this respect, our method draws heavily on the original scheme proposed by Princen, Illingworth, and Kittler.

In this paper, we will investigate the vote scattering arising from errors of the gradient direction $\theta$. The relationship between vote scattering and image size will be delineated. The Hierarchical Peak Compaction Hough Transform (HPCHT) will be proposed. It adapts the parameter-space quantization to the error scale of the line estimates. It thereby subsumes the vote scattering to the grid size of the parameter space. The HPCHT method splits the image into small subimages so that the scattering is controlled within the tolerance. This recursive image partitioning is then followed by a hierarchical subimage merging in which short segments extracted locally are eventually merged to global linear structures. Accuracy is gained in the process and lines extracted at the global level are more accurate. Continuity of line segments across subimages is captured by using overlapped subimage windows. This method fits well with the popular pyramidal approach in multi-resolution image analysis.

The next section will investigate the factors which account for the vote scattering. In Sec. 3, the Hierarchical Peak Compaction Hough Transform will be presented. The issues of vote scattering and line extraction accuracy will be dealt with in Sec. 3.1. Section 3.2 will explain the adaptive parameter-space quantization and a simplified peak-detection algorithm under constrained vote scattering. Experimental results will be presented in Sec. 4. Section 5 concludes the paper.

2. VOTE SCATTERING CAUSED BY NOISY GRADIENT DIRECTION

Consider an image point $P(x, y)$ on an edge line $L$:

$$\rho = x \cos \theta + y \sin \theta,$$

(1)

where $\rho$ is the normal distance from the line $L$ to the origin, and $\theta$ is identical to its gradient direction. Errors in $\theta$ affect the computation for $\rho$. Instead of voting consistently to a sharp peak at $(\theta, \rho)$, an edge point would obtain a $\rho$ deviating from the true value due to the error in $\theta$. This results in a cluster scattering over a wide
range. In real images where many edges are present, the gradient errors would likely cause the clusters to overlap with one another, making interpretation for the vote peaks very difficult. We will show in Sec. 4 with real examples how undefined the vote peaks can be.

Vote scattering can be measured by the estimated error of \( \rho (\Delta \rho) \). Equation (1) indicates that \( \rho \) changes in accordance with \( \theta, x, \) and \( y \). For small \( \Delta \theta, \Delta x, \) and \( \Delta y \),

\[
\Delta \rho \approx \frac{\partial \rho}{\partial \theta} \Delta \theta + \frac{\partial \rho}{\partial x} \Delta x + \frac{\partial \rho}{\partial y} \Delta y = (-x \sin \theta + y \cos \theta) \Delta \theta + \cos \theta \Delta x + \sin \theta \Delta y .
\]

(2)

Usually, \( \Delta x \) and \( \Delta y \), which represent the error of edge localization, can be estimated to pixel precision. Hence, the second and third error terms in Eq. (2) are insignificant. The factor \((-x \sin \theta + y \cos \theta)\) in Eq. (2) will be denoted as \(d\), which measures the distance from the point \(P(x, y)\) to the foot of the normal \(P_0\) (see Fig. 1). Thus, Eq. (2) can be simplified to

\[
\Delta \rho \approx d \Delta \theta .
\]

(3)

If the center of the image is chosen as the origin of the image space, \(d\) could be as large as \((\sqrt{2}/2)N\), where \(N \times N\) is the size of the image. It follows that

\[
\Delta \rho \leq \frac{\sqrt{2}}{2} N \Delta \theta .
\]

(4)

For an image of size 256 \(\times\) 256, if the \(\theta\) values varied with a standard deviation \(\sigma_{\Delta \theta}\) of 5\(^\circ\), \(\Delta \theta\) (which is normally considered lying in a range of \(\pm 3\sigma_{\Delta \theta}\)) would be \(\pm 15\(^\circ\). Then, \(\Delta \rho\) could be as large as \(\pm 48\) pixels. Nearby vote clusters will overlap, and peak detection becomes impossible.

The large \(d\) values in Eq. (3) aggravate the scattering of votes when the edge points are away from the foot of the normal of the underlying line. The effect is illustrated in Fig. 2. Figure 2(a) shows an edge segment of \(\theta = 45^\circ\) and \(\Delta \theta = 5^\circ\) located at the center of the image. When the segment is moved near to the border of the image, as illustrated in Fig. 2(c), the vote cluster spreads out wider in the \(\rho\) dimension (compare Figs. 2(b) and (d)).

Statistically, \(\Delta \theta\) can be modeled approximately as a random variable with zero mean and a standard deviation of \(\sigma_{\Delta \theta}\). For a normal distribution, over 99\% of the samples fall into the range of \(\theta \pm 3\sigma_{\Delta \theta}\). A bound of \(B_{\Delta \theta} = 3\sigma_{\Delta \theta}\) will therefore cover practically all the votes in the \(\theta\) dimension.\(^a\) Substituting \(B_{\Delta \theta}\) for the \(\Delta \theta\) in

\(^a\) For a more general case, a very conservative estimation is given by the Chebyshev's Inequality. Applying it to our problem, it states that for any distribution the probability of having \(|\Delta \theta|\) exceeding an upper bound \(B_{\Delta \theta}\) is \(P(|\Delta \theta| \geq B_{\Delta \theta}) \leq \sigma_{\Delta \theta}^2/B_{\Delta \theta}^2\). This means that at least \(1 - \sigma_{\Delta \theta}^2/B_{\Delta \theta}^2\) of the samples are contained within a range of \(\pm B_{\Delta \theta}\). A bound of \(B_{\Delta \theta} = 3\sigma_{\Delta \theta}\) thus always has an overwhelming probability of covering the samples.
Fig. 1. The geometry of a line $L(\theta, \rho)$ through a point $P(x, y)$ in a 2-D $x$–$y$ plane. $P_0$ is the foot of the normal of $L$. $d$ is the distance of $P$ from $P_0$. $\Delta \theta$ measures the deviation of the $\theta$ estimate at $P$, and $\Delta \rho$ is the error when $\rho$ of $L$ is computed using the inaccurate $\theta$ value at $P$.

Fig. 2. Effects of $d$ for a line segment of $\theta = 45^\circ$ and $\sigma_{\theta} = 5^\circ$. (a) Edge points located near the center of the image. (b) Vote distribution in the $\theta$–$\rho$ space. The votes form a good cluster in the $\rho$ dimension. (c) Edge points near the border of the image. (d) Vote distribution in the $\theta$–$\rho$ space. The votes are more scattered in the $\rho$ dimension compared with (b).
Eq. (4) yields a bound $B_{\Delta \rho}$ on $\Delta \rho$ which will cover practically all the votes in the $\rho$ dimension.

$$B_{\Delta \theta} = 3 \sigma_{\Delta \theta} \quad (5)$$

$$B_{\Delta \rho} = \frac{3 \sqrt{2}}{2} N \sigma_{\Delta \theta} \quad (6)$$

This forms a rectangular region bounded by $\pm B_{\Delta \theta}$ and $\pm B_{\Delta \rho}$ centered at $(\theta, \rho)$, which has a very high probability of containing all the votes in a cluster.

3. THE HIERARCHICAL PEAK COMPACTION HOUGH TRANSFORM (HPCHT)

A Hierarchical Peak Compaction Hough Transform procedure (HPCHT) is proposed. It has two hierarchical processing components. First, the original image is split recursively into small subimages since the $\rho$-scattering gets suppressed in images of small size according to Eq. (6). The splitting continues until the $\rho$-scattering is controlled within a tolerance $\tau$. Second, the Hough transform is applied to each of the small subimages and the results are then merged. As the scattering is controlled within small clusters, the vote peaks can be identified as connected components, and the centroid of a cluster can be used to represent the underlying line segment. A recursive merge procedure is applied to extract long segments by merging the vote peaks of short segments. The segment merging process also follows a Hough transform scheme. At each level, the Hough transform is performed to collect votes by the short segments from the level below. Peaks in the $\theta - \rho$ space indicate the most probable lines at the current level. Again, the line is calculated as the centroid of the connected components. Global linear structures are recognized when the top level is reached.

The result will be more accurate in terms of the $\theta$ and $\rho$ evaluation since a long segment takes the average of the shorter ones. The Hough parameter space is quantized in the scale of the expected vote scattering which is progressively refined. Therefore, vote peaks for long lines at the higher levels can be located more accurately in a Hough space of fine resolution.

A minor issue of consideration is that the actual image subdivision uses an overlapped pyramid where overlap among sibling subimages is provided. Instead of $\frac{1}{2} \times \frac{1}{2}$ at each subdivision, a size ratio of $\frac{3}{5} \times \frac{3}{5}$ is used. The overlap is to capture the continuity of the lines extending across adjacent subimages. The common sub-segment in the overlapping region helps to ensure agreement on the underlying extended line when two segments merge from adjacent subimages.

In summary, the HPCHT procedure is as follows:

1. **Scattering Test**: Calculate the scattering, $B_{\Delta \theta, \text{out}}$ and $B_{\Delta \rho, \text{out}}$, of the current image using Eqs. (5) and (6) with the current image size $N_i$ and $\sigma_{\Delta \theta, \text{out}}$.

2. **Recursive Subdivision**: If $B_{\Delta \text{out}}$ is larger than the tolerance $\tau$, split the image into
four subimages with a size ratio of $\frac{3}{4} \times \frac{3}{4}$. Repeat Step (1) and Step (2) recursively until $B_{\Delta \rho_{init}} \leq \tau$.

3. **Hough Transform**: Apply the Hough Transform independently to each of the current subimages. For each subimage, map its edge segments to the parameter space according to the $\theta_{i-1}$ derived from the level below. The Hough parameter space is quantized using the grid size, $h_{\theta} \times h_{\rho}$, which is calculated from the scattering $B_{\Delta \theta_{i-1}}$ and $B_{\Delta \rho_{i-1}}$ using Eqs. (11)-(14) in Sec. 3.2.1. Initially, a segment consists of a single edge point with $\theta_{init}$ obtained from the edge-detection process. The initial $B_{\Delta \theta_{init}}$ and $B_{\Delta \rho_{init}}$ are obtained from Step (1).

4. **Peak Detection**: Collect the votes for a peak as a connected component in the Hough space. The centroid of the peak gives the $(\theta_i, \rho_i)$ value of the underlying line. The contributing edge segments are now represented by the line as a group sharing the same $\theta_i$ and $\rho_i$ values. The errors $\sigma_{\Delta \theta_i}$ and $\sigma_{\Delta \rho_i}$ are improved through the averaging process.

5. **Recursive Merging**: Go up one level to the parent subimage. Perform Step (3) through Step (5) recursively to extract the lines underlying the segments until it reaches the top of the hierarchy.

6. Report the lines extracted.

3.1. Accuracy of the Line Extraction

Initially, the $\Delta \rho$ error is kept within the tolerance by partitioning the image into small subimages. As line segments merge, the $\theta$ and $\rho$ estimates of the resulting line are refined. Consequently, $\Delta \rho$ is always kept small even when small subimages merge to form larger ones.

3.1.1. Precision of the $\theta$ estimates

The position of the peak is estimated by simple averaging on the $\theta$ and $\rho$ values, respectively, of all the votes within the cluster. The result is a better estimate of the $(\theta_i, \rho_i)$ value of the underlying line. To illustrate this, let $\Delta \theta_1, \ldots, \Delta \theta_m$ be the $\Delta \theta's$ of $m$ collinear image points. Suppose they are independent random variables with the same probability distribution. Let the standard deviation of the distribution be $\sigma$. The average of the $m$ $\Delta \theta's$ also has a random distribution but with an improved standard deviation. The factor of improvement is $\sqrt{m}$. Therefore, as longer segments are formed in the hierarchy, the statistical error in the resulted $\theta$ estimates would be reduced accordingly. At level $i$, the $\sigma_{\Delta \theta_i}$ would be smaller than the initial estimate $\sigma_{\Delta \theta_{init}}$, by an order of $\sqrt{N_i}$, where $N_i \times N_i$ is the size of the subimage at level $i$.

$$
\sigma_{\Delta \theta_i} = \frac{\sigma_{\Delta \theta_{init}}}{\sqrt{N_i}}.
$$

3.1.2. Accuracy of the $\rho$ values

According to Eq. (6), the $\rho$-scattering $B_{\Delta \rho}$ grows linearly with the image size $N$. However, with $\sigma_{\Delta \theta}$ being reduced substantially from level to level, $B_{\Delta \rho}$ is controlled
at small magnitudes. In the following, the errors of the \( \rho \) values for different levels are derived.

As shown in Eq. (3) and Fig. 2, vote clusters are less scattered for the edge points with smaller \( d \) values. In Steps (1) and (2) of the HPCHT procedure, the image is partitioned recursively until \( \Delta \rho \) is contained within the \( \rho \)-scattering tolerance \( \tau \). The spatial separation between parallel lines\(^b\) in the original image corresponds to the inter-cluster separation in the Hough parameter space, and hence represents a good criterion for the choice of \( \tau \). For the cube image in Fig. 6(a), \( \tau \) is chosen as six pixels. If \( \sigma_{\Delta \rho} \) is 5°, then

\[
B_{\Delta \rho_{\text{out}}} = \frac{3\sqrt{2}}{2} N_0 \cdot \frac{5}{180} \pi \leq \tau = 6
\]

\[
N_0 \leq 32
\]

It means that a subimage at level 0 of size no larger than 32 \( \times \) 32 pixels will always have its vote clusters centralized within ±6 pixels along the \( \rho \)-axis. At that scale of scattering, the vote clusters are compact and well separated. Cluster isolation and peak localization become straightforward. A simple strategy for peak detection will be discussed in the next section.

In Steps (3) to (5) of the HPCHT procedure, the segments are merged recursively from bottom up. At level \( i \) four child subimages of size \( N_{i-1} \times N_{i-1} \) are merged to form the current subimage of \( N_i \times N_i \). The \( \rho \) values of the child segments are recalculated with reference to the coordinate system of the current subimage. The calculation essentially involves a translation of the origin of the child space to the current space.

\[
\rho_i = \rho_{i-1} + x_{0_{i-1}} \cos \theta_{i-1} + y_{0_{i-1}} \sin \theta_{i-1}
\]

(9)

where \( \theta_{i-1} \), \( \rho_{i-1} \), \( \rho_i \) are the \( \theta \) and \( \rho \) values for the two levels respectively; \((x_{0_{i-1}}, y_{0_{i-1}})\) is the position of the child space's origin measured in the current space's coordinate system (see Fig. 3).

Similar to the derivation of \( \Delta \rho \) in Eq. (2), \( \Delta \rho_i \) can be derived from Eq. (9) as:

\[
\Delta \rho_i = \Delta \rho_{i-1} + \frac{\sqrt{2}}{2} N_{i-1} \Delta \theta_{i-1}
\]

With \( \Delta \theta_{i-1} \) and \( \Delta \rho_{i-1} \) being considered as independent random variables, \( \Delta \rho_i \) is treated as a random variable as well. The average of the \( \Delta \rho_i \)’s among the child segments has a variance which is half of the original value (since the length of the segment is assumed to be doubled in the merging process).

\(^b\) Note that parallel lines are lines with the same \( \theta \) value. Opposite edges of an image pattern are anti-parallel, i.e. their \( \theta \) values differ by \( 180^\circ \).
Fig. 3. The $\theta$ and $\rho$ values of the child segments are recalculated with reference to the current space's coordinate system during the merging process. It essentially involves a translation of the origin of the child space to the current space. $(x_{0i}, y_{0i})$ and $(x_{0i-1}, y_{0i-1})$ are the origins of the image spaces at levels $i$ and $i-1$ respectively; $\theta_i$ and $\theta_{i-1}$ are the $\theta$ estimates; $\rho_i$ and $\rho_{i-1}$ are the $\rho$ estimates and $d$ measures the translational displacement of the foot of the normal.

Fig. 4. The trend of $\sigma_{\theta\omega}$ at different $\tau$ values when the image size $N_i$ increases up the subimage hierarchy. The initial $\sigma_{\theta\omega}$ is $5^\circ$. 
\[
\sigma_{\Delta \rho_i}^2 \approx \frac{1}{2} \left( \sigma_{\Delta \rho_{i-1}}^2 + \frac{1}{2} N_i^2 \sigma_{\Delta \theta_{i-1}}^2 \right).
\]

Take \( \sigma_{\Delta \rho_i} = \frac{\sigma_{\Delta \rho_{\max}}}{\sqrt{N_0}} \), and \( \sigma_{\Delta \rho_{\max}} = \frac{B_{\Delta \rho_{\max}}}{3} = \frac{\tau}{3} \). The recurrence relation yields

\[
\sigma_{\Delta \rho_i} \approx \left( \frac{\tau^2}{9} \frac{1}{N_i} + \frac{\sigma_{\Delta \theta_{\max}}^2}{6} N_i \right)^{1/2}.
\] (10)

Figure 4 shows that the growth rate of \( \sigma_{\Delta \rho_i} \) in Eq. (10) is much better than the linear growth rate which is described in Eq. (6) for the standard Hough transform. From the figure, \( \sigma_{\Delta \rho} \), stays small at the small \( N_i \) values. As \( N_i \) increases, the second component inside the square root of Eq. (10) dominates, and \( \sigma_{\Delta \rho_i} \) grows with \( \sqrt{N_i} \).

In one of our experiments, an image of 256 \times 256 with \( \sigma_{\Delta \theta_{\max}} = 5^\circ \) and \( \tau = 6 \) pixels (as in Fig. 6(a)) was used. The \( \rho \)-scattering \( \sigma_{\Delta \rho} \) did not exceed \( \pm 1 \) pixel at the top level of the hierarchy. A practical bound \( B_{\Delta \rho} \) of \( \pm 3 \) pixels was sufficient to contain the votes in good clusters.

3.2. Peak Detection in the Low Scattering Hough Space

For a distribution with separable clusters, a feasible peak-detection method is to simply isolate individual clusters. The centroid of each cluster will give a good statistical mean value for the peak. The question is: how do we guarantee the separability of the clusters and yet avoid spurious splitting at the same time? Normally, the Hough space is implemented as a discrete space of accumulator cells. The grid size has to be tuned carefully to achieve both the cluster separability and cluster connectivity. Grid sizes that are too coarse will cause cluster collision; grid sizes that are too fine will split a cluster. In the HPCHT, the problem of quantization is solved by using an adaptive scale that guarantees a low scattering of votes. Moreover, the quantization is refined from level to level to achieve a Hough transform of high accuracy.

3.2.1. Quantization of the \( \theta-\rho \) space

According to Sec. 3.1, there is a gain in the \( \theta \)-precision from level to level, while the \( \rho \)-scattering remains low. As a result, when subimages merge, collinear segments extracted from different subimages are expected to vote to a peak which is more compact than the peaks obtained at the level below. To take advantage of the precision gain, quantization of the Hough space is refined accordingly to increase the resolution of the line extraction. Hence, the grid size \( h_{\theta_i} \) and \( h_{\rho_i} \) are chosen as:

\[
h_{\theta_i} = 3\sigma_{\Delta \theta_i},
\] (11)

\[
h_{\rho_i} = 3\sigma_{\Delta \rho_i}.
\] (12)
At level 0, the $\sigma_{\Delta \theta_{\text{int}}}$ values are estimated from the edge data, and the $\rho$-scattering is $\tau$, which is determined based on the spatial separation of the parallel lines in the original image.

$$h_{\theta_0} = 3\sigma_{\Delta \theta_{\text{int}}}$$  \hspace{1cm} (13)

$$h_{\rho_0} = \tau.$$  \hspace{1cm} (14)

3.2.2. Peak detection by finding connected components

As the grid size is adapted to the vote scattering, a vote cluster is either contained in a single grid cell or falls into two cells when it resides on the grid line. In any case, since each individual vote cluster spreads over no more than two neighboring cells, the peak-detection process is straightforward. It only needs to inspect the adjacent $\theta-\rho$ cells to discover the connected components. Each of these isolated connected components stands for a vote peak, and its position can be determined by the average of all the contributing votes.

4. EXPERIMENTAL RESULTS

The proposed HPCHT has been implemented and applied on various real images. Our results show that the HPCHT greatly improved the performance of line detection in all test cases.

Figure 5(a) shows the 256 $\times$ 256 image of two Rubik cubes. Figure 5(b) is a 512 $\times$ 512 image of an office scene. Figure 6 shows the edge maps of the Rubik

![Fig. 5. Two examples used in the experiments. (a) A 256 $\times$ 256 image of two Rubik cubes. (b) An office scene. The size is 512 $\times$ 512.](image-url)
cubes and the office scene respectively. A set of $3 \times 3$ Sobel-like edge marks is used to calculate the edge location and orientation. The resulted edge image is then filtered with a non-maximum suppression method to obtain thin edges. It is observed that $\sigma_{\Delta \theta}$ is $5^\circ$ for the cube image and $6^\circ$ for the office scene. These numbers are taken as our initial $\sigma_{\Delta \theta_{\text{mit}}}$.

The ordinary Hough transform methods easily fail in the presence of severe errors in the $\theta$ values. As indicated by Eq. (3) in Sec. 2, errors in $\theta$ are manifested in $\rho$-scattering of the Hough voting. Figure 7 clearly shows that the errors in $\theta$ and $\rho$ widely scatter the votes in the Hough space in both $\theta$ and $\rho$ directions. Although some peaks are still observable, their positions are difficult to evaluate. In Fig. 7(b), the $\rho$-scattering is so severe that multiple peaks are practically fused together even though the Hough space is quantized at a fine scale of $1^\circ$ and 3 pixels.

Fig. 6. Edge maps for the images in Fig. 5. (a) The Rubik cubes, $\sigma_{\Delta \theta} = 5^\circ$. (b) The office scene, $\sigma_{\Delta \theta} = 6^\circ$.

Fig. 7. The vote distribution from the standard Hough transform on the two edge images. (a) The Rubik cubes with a Hough space quantization of $1^\circ$ and three pixels. (b) The office scene with a Hough space quantization of $1^\circ$ and three pixels.
Our method starts with small subimages where \( B_{\Delta \rho_{\text{int}}} \leq \tau \), i.e. the \( \rho \)-scattering is reduced to within the tolerance allowed by the spatial separation of the parallel lines. As observed in Fig. 6, the edge distribution of the cube image is sparse whereas the office scene has a much denser edge map. Parallel edges are more than 12 pixels away from each other in the former image but as close as six pixels in the latter. In our experiments, \( \tau \) was chosen to be six pixels for the cube image, and three pixels for the office scene. The HPCHT splits the image from the top down in the subimage hierarchy and then merges the segments extracted from the bottom up.

Figure 8 shows the vote peaks at different levels of the hierarchy of the test image in Fig. 6(a). The hierarchy is implemented as an overlapped pyramid. With \( \sigma_{\Delta \theta_{\text{int}}} = 5^\circ \), the cube image is partitioned from the original size \( N_4 = 256 \) into \( N_3 = 160 \), \( N_2 = 100 \), \( N_1 = 62 \) and then \( N_0 = 38 \). The vote scattering (\( \sigma_{\Delta \theta} \) and \( \sigma_{\Delta \rho} \)) at each level is calculated using Eqs. (7) and (10). When the grid size (\( h_{\theta} \) and \( h_{\rho} \)) is chosen on the same scale based on Eqs. (11) to (14), the vote scattering becomes subdued to the grid size. Note that the accuracy of the extracted lines

![Diagram](image)

Fig. 8. The vote peaks at different levels of the subimage hierarchy of the Rubik cube image in Fig. 6(a). (a) Level 0 (from a 38 × 38 subimage at the bottom level). With \( \sigma_{\Delta \theta_{\text{int}}} = 5^\circ \) and \( \sigma_{\Delta \rho_{\text{int}}} = 2 \) pixels, the grid size is chosen to be \( h_{\theta_0} = 15^\circ \) and \( h_{\rho_0} = 6 \) pixels. (b) Level 1 (from a 62 × 62 subimage). The grid size is \( h_{\theta_1} = 2^\circ \) and \( h_{\rho_1} = 1.1 \) pixels. (c) Level 2 (from a 100 × 100 subimage). The grid size is \( h_{\theta_2} = 1.5^\circ \) and \( h_{\rho_2} = 1.2 \) pixels. (d) Level 3 (from a 160 × 160 subimage). The grid size is \( h_{\theta_3} = 1.2^\circ \) and \( h_{\rho_3} = 1.4 \) pixels. (e) Level 4 (from the entire 256 × 256 image). The grid size is \( h_{\theta_4} = 1^\circ \) and \( h_{\rho_4} = 1.8 \) pixels. The grid size of the Hough space is chosen at the same scale of the line accuracy so that the vote scattering is subdued to the grid size.
Fig. 9. The vote peaks of the test image Fig. 6(b). At the top level of the hierarchy, the Hough space is quantized with a grid size of \( h_\theta = 1^\circ \) and \( h_\rho = 3.5 \) pixels.

Fig. 10. The line images reconstructed from the results of the HPCHT to illustrate the accuracy of the method. (a) The Rubik cubes. (b) The office scene.

increases from level to level while the vote scattering is limited to adjacent grids in the Hough space. A similar result is also achieved from the test image in Fig. 6(b). Figure 9 shows the vote peaks at the top level. An accuracy of about \( \pm 1^\circ \) and \( \pm 3 \) pixels is achieved on both test images. The ambiguity problem illustrated in Fig. 7 vanishes in both Figs. 8 and 9. Peak detection is straightforward in the latter case.

The accuracy of the HPCHT line extraction algorithm is illustrated with the reconstructed line images in Fig. 10. Comparing the result with the edge maps in Fig. 6, we see that practically all the long edges have been extracted successfully. Note that the line orientation \( \theta \) and the position \( \rho \) have been accurately recovered.
5. CONCLUSION

This paper pointed out that error in the gradient direction is the major factor contributing to vote scattering in the Hough parameter space. Following the observation that the variance of vote scattering is affected by the location of the point of origin, we partition the image into small regions to constrain the vote scattering. A Hierarchical Peak Compaction Hough Transform (HPCHT) is proposed, which uses the Hough transform procedure on small subimages initially, and a recursive Hough transform merging scheme on the extracted line segments in the subsequent steps. A bound on vote scattering has been derived which guides the recursive subdivision of the image in the first stage, and the adaptive quantization of the parameter space in the second stage of the algorithm. As a result, an accurate Hough transform of low $\rho$-scattering and high $\theta$-precision has been achieved. Experimental results from the recursive HPCHT algorithm have been shown to confirm our analysis.

Princen, Illingworth and Kittler’s Hierarchical Hough Transform (HHT)\textsuperscript{23} represents the recent result in hierarchical line finders. Our method draws heavily on the grouping mechanism of the HHT for line detection. However, the main differences between our HPCHT and Princen et al.’s work can be summarized as follows. First, the image partitioning in HPCHT is driven by the error scale of the gradient directions. The depth of the pyramid is chosen to ensure that the subimages are sufficiently small so that the vote scattering is confined to adjacent grids of the parameter space. Second, the quantization of the parameter space is also related to the error scale of the line estimates, rather than the discretization of the image space. As a result, a non-scattered vote accumulation is always obtained and the multiple-votes accumulation for accommodating the uncertainty in the vote position is unnecessary. Third, since the peaks are sharp and distinct, the peak finding problem can be simplified. There is no need to introduce the reaccumulating strategy for peak extraction. Therefore, we believe that our method is capable of adapting to the error scale of the edge data and making use of a simple and reliable peak detection method for line extraction.

As in Princen’s HHT, the hierarchical structure of the recursive line extraction facilitates independent execution of the Hough procedures on individual local subimages. Therefore, the HPCHT is also suitable for fast parallel implementation on pyramid computers.

ACKNOWLEDGEMENTS

We would like to thank Bob Laughlin and Kevin Tate for their helpful comments to this manuscript. This work is supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) under grant A-36726.
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Received 19 April 1991; revised 11 December 1991.

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