Data Representation: 2’s Compliment
Reminders

Liaqat Ali, Summer 2018.
One-Stop Access To Course Information

- **Course website**: One-stop access to all course information.
  
  [http://www2.cs.sfu.ca/CourseCentral/120/liaqata/WebSite/index.html](http://www2.cs.sfu.ca/CourseCentral/120/liaqata/WebSite/index.html)

  - Course Outline
  - Exam Schedule
  - Python Info
  - [CourSys/Canvas](#) link
  - Learning Outcomes
  - Office Hours
  - Textbook links
  - and more...
  - Grading Scheme
  - Lab/Tutorial Info
  - Assignments

- **Canvas**: Discussions forum - [https://canvas.sfu.ca/courses/39187](https://canvas.sfu.ca/courses/39187)

- **CourSys**: Assignments submission, grades - [www.coursys.sfu.ca](http://www.coursys.sfu.ca)

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How to Learn in This Course?

A - Attend Lectures & Labs
R - Read / review Textbook/Slides/Notes
R - Reflect and ask Questions
O - Organize — your learning activities on weekly basis, and finally...
W - Write Code, Write Code, and Write Code.

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Deliverables

1. Deliverables are due by the given date and time.
2. For the course, we are using IDLE to write and run our Python code.
3. You can use the CSIL lab computers outside your lab hours.
4. Plan ahead your assignments and other deliverables. Computer crash, network problems etc. are not acceptable excuses for delays in deliverables.
5. You may use online Python interpreters for running and testing your codes, such as:
   https://repl.it/languages/Python3

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Labs

1. Each lab has an assigned TA.
2. Attend your assigned lab and show your work to your TA for the participation marks.
3. Class enrolments and lab swaps are closed now.
Course Topics

1. General introduction
2. Algorithms, flow charts and pseudocode
3. Procedural programming in Python
4. Data types and Control Structures
5. Fundamental algorithms
6. Binary encodings
7. Basics of computability and complexity
8. Basics of Recursion
9. Subject to time availability:
   □ Basics of Data File management
Today’s Topics

• Data Representation (Binary Encoding)
  1. Unsigned Integer
  2. Signed Integer
  3. Binary Addition
  4. 1’s Compliment Representation
  5. 2’s Compliment Representation
Data Representation: 2’s Compliment

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Two’s Complement Signed Integer Representation

- **Integer is represented by a string of binary digits.**
  - Representation is in 2’s compliment form.
  - Right most bit is used for sign.
  - Remaining bits represent the value.

- **Decimal to 2’s Compliment form:**
- **For a Positive Number:**
  1. First bit is 0.
  2. Convert the number to its binary equivalent.
- +7 is represented as: 0000 0111
- +13 is represented as: 0000 1101

- **For a Negative Number:**
  1. Convert the number to its binary equivalent.
  2. Flip the bits
  3. Add 1.
- -7 would be represented as:
  1. Convert to binary: 0000 0111
  2. Flip the bits: 1111 1000
  3. Add 1. 1 = 1111 1001
- -13 would be represented as:
  1. Convert to binary: 0000 1101
  2. Flip the bits: 1111 0010
  3. Add 1. 1 = 1111 0011

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Two’s Complement Signed Integer Representation - 2

- **2’s Compliment to Decimal:**
  - **If first bit is 0, then:**
    1. The number is positive.
    2. Simply, convert the binary number to its decimal equivalent.
  - **0001 0111** is 2’s compliment representation of: \( +2^4 + 2^2 + 2^1 + 2^0 = +23 \)
  - **If first bit is 1, then:**
    - The number is negative.
    - Flip all the bits. So, **1011 0001** becomes **0100 1110**
    - Add 1. \( 1 = 0100 1111 \)
    - Convert to decimal: **0100 1111** = \( 2^6 + 2^3 + 2^2 + 2^1 + 2^0 = 64 + 8 + 4 + 2 + 1 = 79 \)
    - So **1011001** represents **-79**
Two’s Complement Signed Integer Representation - 3

- **2’s Compliment to Decimal:**
  - **0000 0000** is a 2’s compliment representation of which decimal number?
    1. First bit is 0, so this is a representation of a positive number.
    2. Convert the bits to the decimal equivalent. +0 = 0
  - **1000 0000** is a 2’s compliment representation of which decimal number?
    1. First bit is 1, so this is a representation of a negative number.
    2. Flip all the bits. So, **1000 0000** becomes **0111 1111**
    3. Add 1. 1 = **1000 0000**
    4. Convert to decimal: **1000 0000** = \(2^7 = 128\)
    5. So **1000 0000** represents **-128**
- So, in 2’s compliment, we no longer get two representations of 0.
More Examples: Two’s Complement to Decimal

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<th>1</th>
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-85

35

Remember if first digit is 1 flip bits then add 1

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Adapted from: Janice Regan, 2013.
More Examples: Two’s Complement to Decimal - 2

Remember if first digit is 1 flip bits then add 1

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<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Adapted from: Janice Regan, 2013.
More Examples: Decimal to 2’s Complement

• -72  (number < 0)
  ▫ Express 72 in 8 bit binary
    • 64+8
    • 01001000
  ▫ Flip the bits:
    • 10110111
  ▫ Add 1:
    • 10111000

• 35  (number > 0)
  ▫ Express 35 in 8 bit binary
    • 32+2+1
    • 00100011
Your Turn

• Which number is represented by the following 2’s compliment pattern?
  1. 10101010
  2. 11011010

Represent in two’s complement form.
  1. 120
  2. -59

Adapted from: Janice Regan, 2013.
### Compare Representations

<table>
<thead>
<tr>
<th>Bit Pattern</th>
<th>Decimal Value in Unsigned Representation</th>
<th>Decimal Value in Signed Representation</th>
<th>Decimal Value in 1’s Comp Rep.</th>
<th>Decimal Value in 2’s Comp Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000 0000</td>
<td>0</td>
<td>+0</td>
<td>+0</td>
<td>0</td>
</tr>
<tr>
<td>0000 0001</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0000 0010</td>
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<td>2</td>
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<tr>
<td>0111 1110</td>
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<tr>
<td>0111 1111</td>
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<td>127</td>
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<tr>
<td>1000 0000</td>
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<td>-127</td>
<td>-128</td>
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<tr>
<td>1111 1111</td>
<td>255</td>
<td>-127</td>
<td>-0</td>
<td>-1</td>
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</table>

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### Twos Complement Addition

<table>
<thead>
<tr>
<th>0 0 1 1 1 0 0 0</th>
<th>56</th>
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</thead>
<tbody>
<tr>
<td>0 1 0 0 0 0 0 0</td>
<td>64</td>
</tr>
<tr>
<td>----------------</td>
<td>------</td>
</tr>
<tr>
<td><strong>0 1 1 1 1 0 0 0</strong></td>
<td><strong>+120</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0 0 1 1 1 0 1 1</th>
<th>59</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 0 0 0 1 1 1</td>
<td>−121</td>
</tr>
<tr>
<td>----------------</td>
<td>------</td>
</tr>
<tr>
<td><strong>1 1 0 0 0 0 1 0</strong></td>
<td><strong>−62</strong></td>
</tr>
</tbody>
</table>

Adapted from: Janice Regan, 2013.
Twos Complement Addition

Adapted from: Janice Regan, 2013.
Twos Complement Overflow

\[
\begin{array}{cccccccc}
0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 1
\end{array}
\begin{array}{c}
110 \\
91 \\
-55
\end{array}
\text{(sum exceeds +127)}
\]

\[
\begin{array}{cccccccc}
1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1
\end{array}
\begin{array}{c}
-96 \\
-59 \\
\text{(sum exceeds -128)}
\end{array}
\]

\[
\begin{array}{cccccccc}
1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1
\end{array}
\begin{array}{c}
+99
\end{array}
\]

Adapted from: Janice Regan, 2013.
Overflow: 2s Complement

- If the sum of two positive numbers is negative, overflow has occurred.
- If the sum of two negative numbers is positive, overflow has occurred.
- Overflow does not occur adding a positive number and a negative number.
- Overflow happens when there is carry over into the sign bit.

Adapted from: Janice Regan, 2013.
2’s Complement

- Multiplication is performed by repeated addition in 2’s complement form.
- Division is performed by repeated subtraction in 2’s complement form.
Your turn again

- -66 : Represent as 2’s compliment.
- 32 : Represent as 2’s compliment.

- 48 – 64 : Perform 2’s compliment addition.
- 57 + 22 : Perform 2’s compliment addition.
Class Participation: Canvas Post

• How would computer add the following two numbers using twos compliment?
  
  +65  
  -23

Required:
1. Write +65 as a 2’s Compliment number.
2. Write -23 as a 2’s Compliment number.
3. Add both the numbers
4. Post your solution on Canvas by tonight, Monday 11:59pm.
Questions?